# Lesson 10

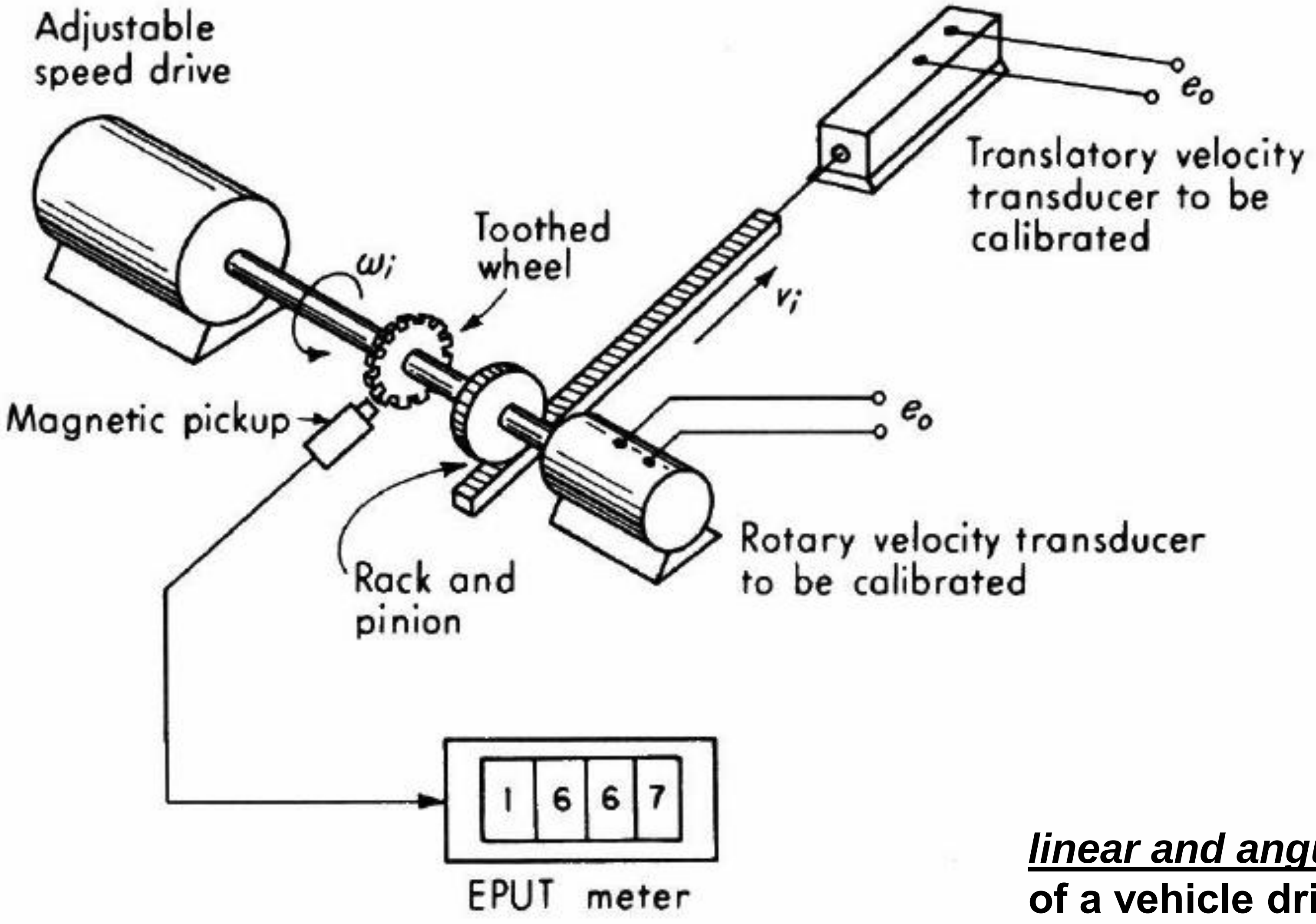
# Thermomechanical Measurements for Energy Systems (MENR)

# Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

*Zaccaria (Rino ) Del Prete*

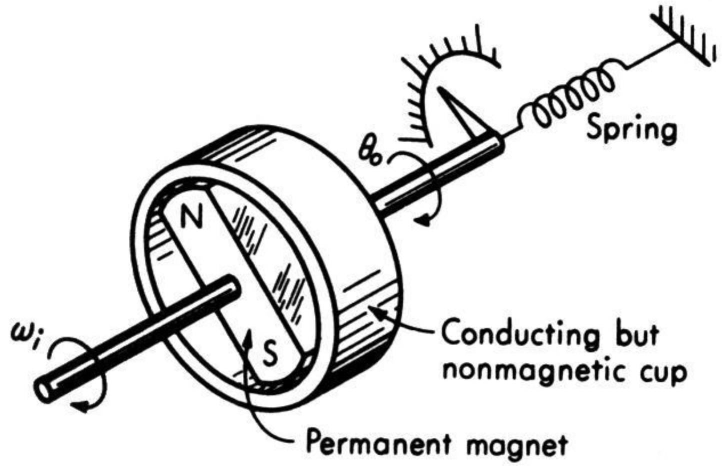
# Some VELOCITY measurement techniques …

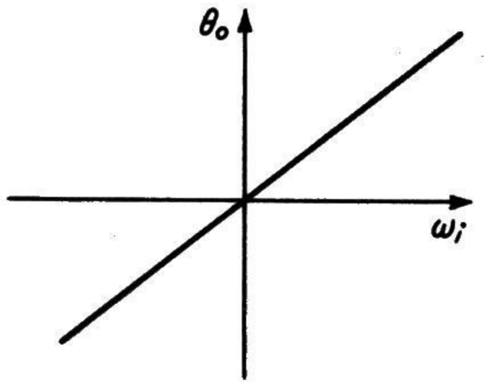



Velocity-calibration setup.

***linear and angular velocities***
**of a vehicle driving shaft …**

# *Speedometer*




There is "no contact" between the *rotating shaft* with the permanent magnet and the *shaft* with the *indicator*. Functioning is based on the <u>*eddy currents*</u> the magnet induces in the cup !

The fundamental starting law is: $F = Bli$ There is actually a "pair of braking forces" just under the rotating poles: $C = F \cdot b = Bil \cdot b$

The electromotive force is $e = Blv$ with $v = \omega \cdot b/2$

The eddy currents then can be globally written as $i = \frac{e}{R} = \frac{Blv}{R} = \frac{Blb \cdot \omega}{2R}$

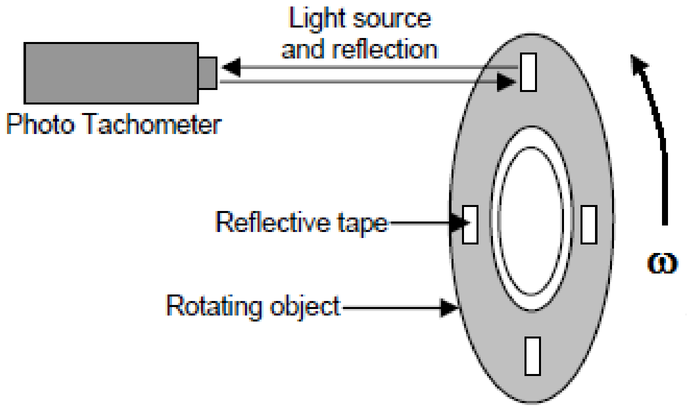
When we substitute this equation into that of the "couple of braking forces" we get: $C = Blb \cdot \frac{Blb \cdot \omega}{2R} = \frac{(Blb)^2 \cdot \omega}{2R}$ which must be equal to the "braking couple" $C_e = k\theta$ of the indicator shaft !

For every angular velocity $\omega$ there is an equilibrium between force couples $C = C_e$ which results in …

$\frac{(Blb)^2 \cdot \omega}{2R} = k \cdot \theta$ and we get a *linear graduation curve* (shown in the figure):

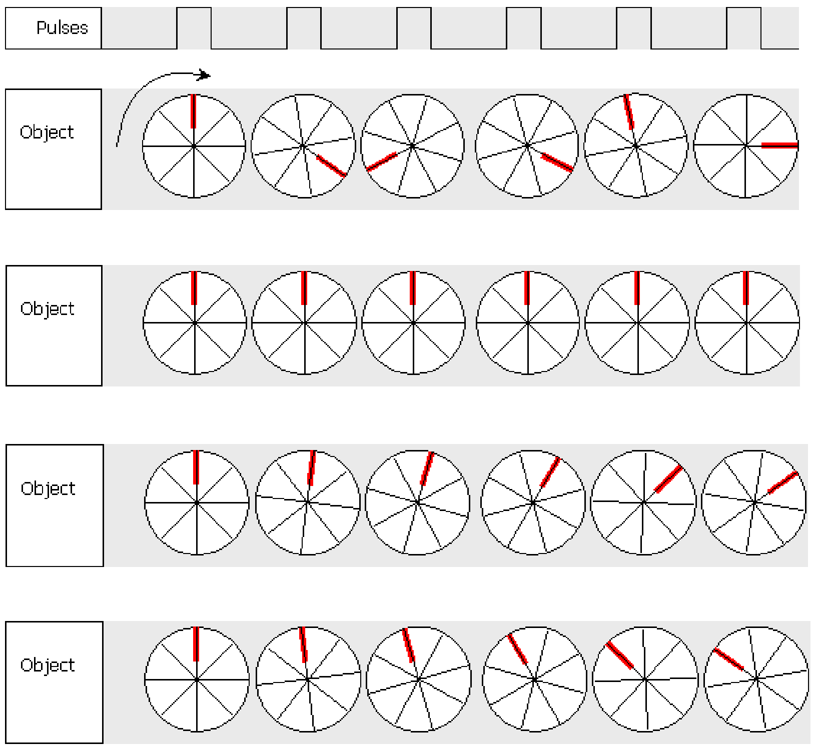
$$\theta = \frac{(Blb)^2}{2Rk} \cdot \omega$$

# *Stroboscope meter*




It measures the *angular speed* $\omega$ of a shaft or a disc without contact, exploiting the persistence time of the light on the human retina (*1/30 s*) !

The instrument has a ***flashing light source*** a and a ***light reading device***; one (or more) reflective mark is applied on the rotating object, the *light flashing frequency* $\boldsymbol{f_g}$ needs to be "tuned" with the *disk rotating frequency* $\boldsymbol{\omega}$ …



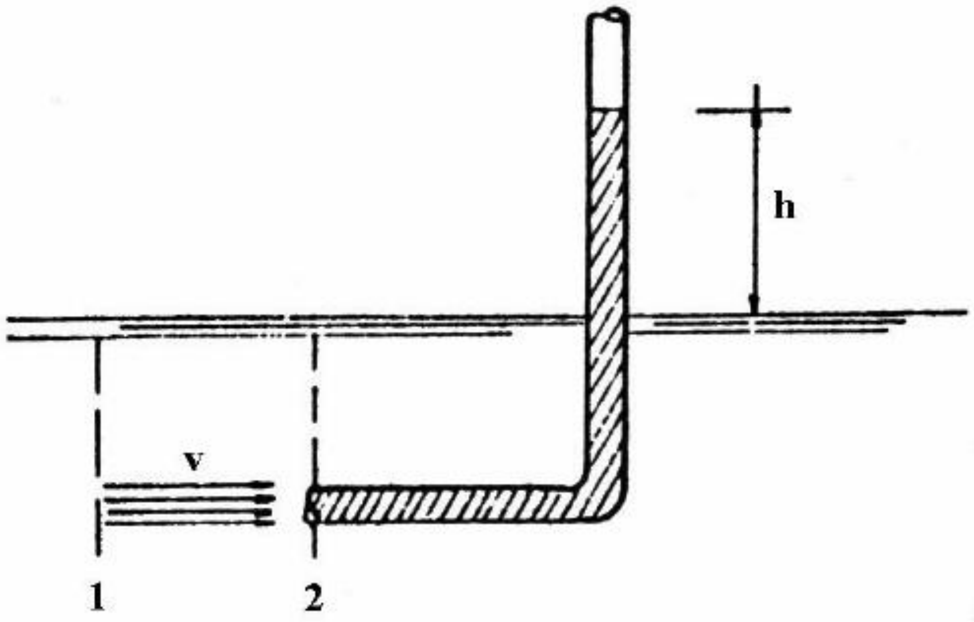

The oscillator is set to a low frequency to start with, the mark will appear at random points around the object !

The frequency of the oscillator $f_g$ is then gradually increased until the mark appears to remain stationary: $\omega = 2\pi f_g$

If the strobe frequency $f_g$ is slightly under the speed of the disk $\omega$, the mark will creep forward /

If the strobe frequency $f_g$ is slightly faster than the speed of the disk $\omega$, the mark will creep forward !

# *VELOCITY measurement in fluids*




Section (1) $E = p_{st} + \frac{1}{2}\rho v^2$ *pressure energy + kinetic energy*

Section (2) $E = p_{TOT}$ *<u>total pressure energy</u>* (called *stagnation*)

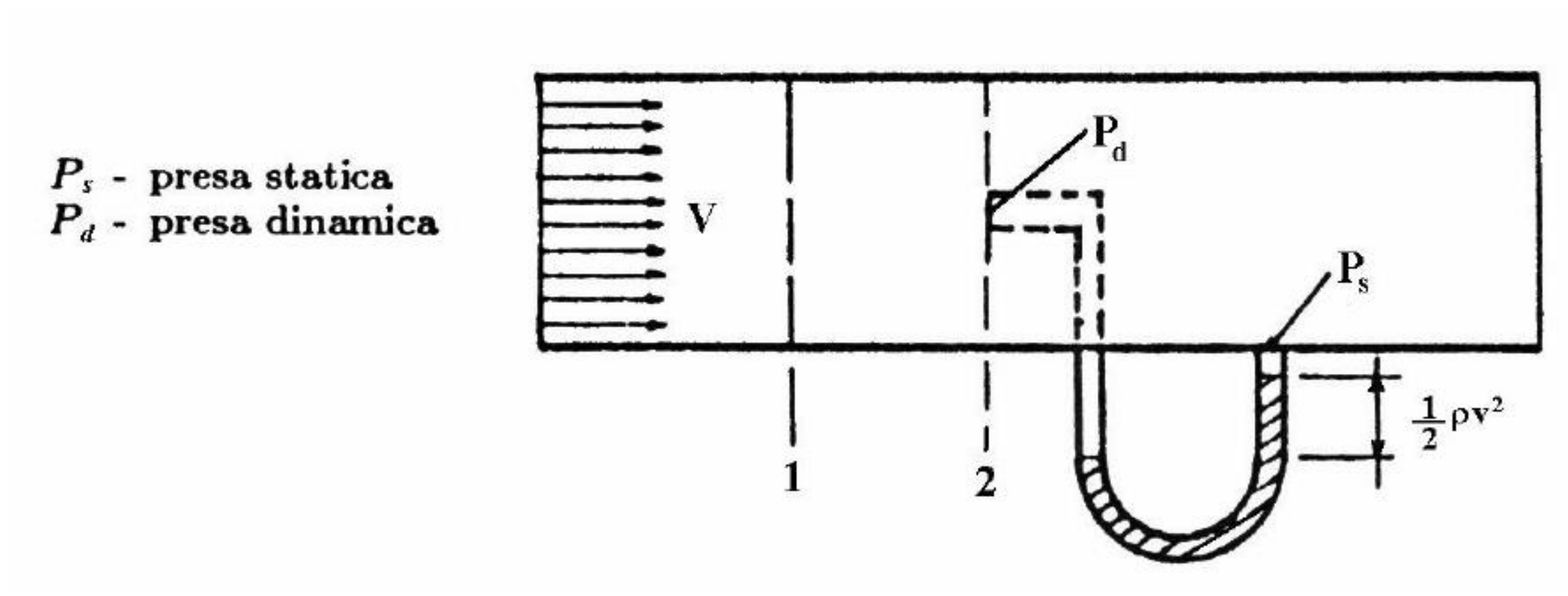
The energy of the fluid is kept between sections (1) and (2) therefore, we can write the ***Bernoulli's equation***:

$$p_{st} + \frac{1}{2}\rho v^2 = p_{TOT}$$

If in equation $E = p_{st} + \frac{1}{2}\rho v^2$ we substitute the ***Stevino's law*** $p_{TOT} - p_{st} = \rho g h$ we get …

the first *graduation curve*: $\frac{1}{2}\rho v^2 = \rho g h$ → $h = \frac{1}{2g} v^2$ which, however, is NOT linear !

With a little improvement, this simple device can be employed also for ***gas*** flowing inside bigger pipes …

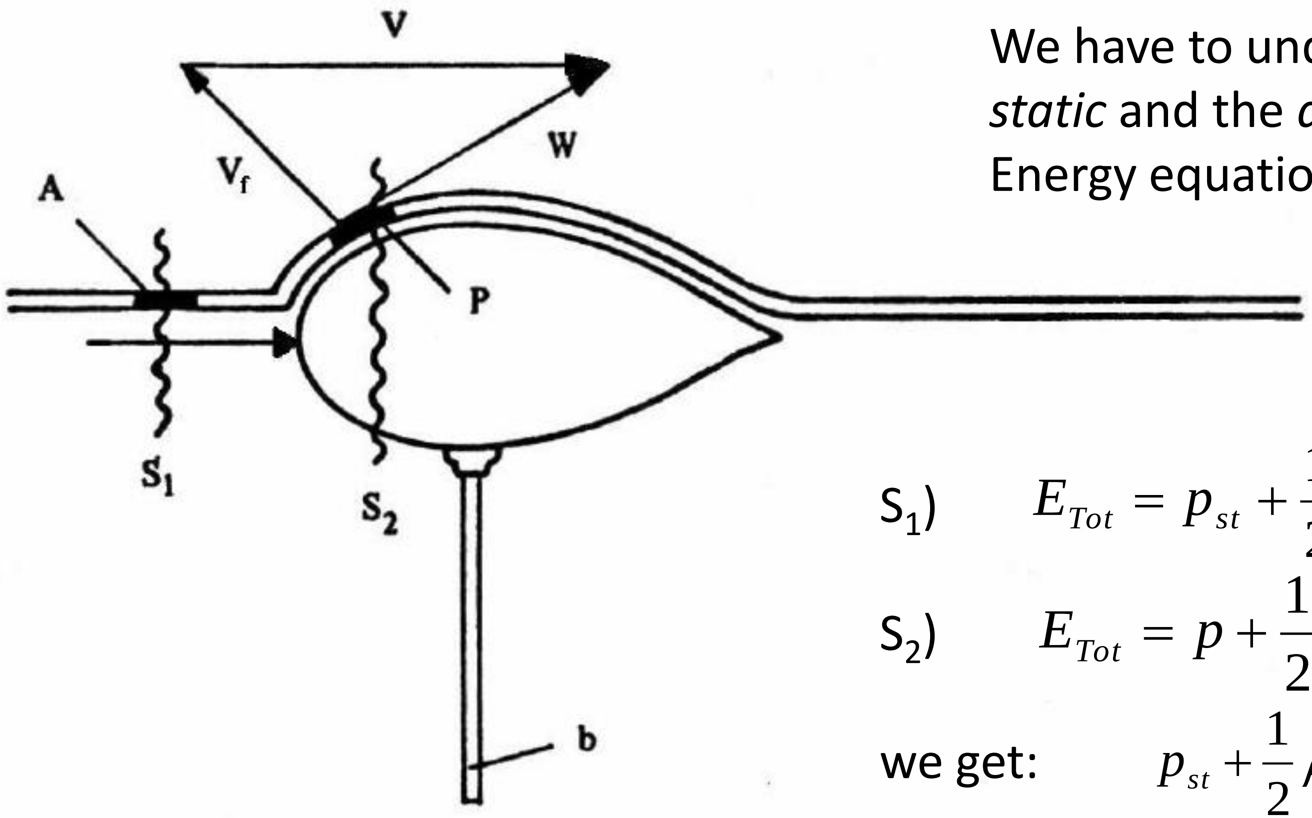



There are <u>two ports</u> that can be seen in the figure: the ***dynamic port $P_d$*** and the ***static port $P_s$***

Note that here, the density of the fluid which flows in the pipe must be different (much *smaller*) than the density of the manometer fluid: $\rho \neq \rho^*$ the *graduation curve* is therefore …

$$v = \sqrt{\frac{2 \cdot \Delta P}{\rho}} = \sqrt{\frac{2 \cdot \rho^* gh}{\rho}}$$

# *VELOCITY measurement for fluids flowing NOT in pipes*




We have to understand where to correctly position the *static* and the *dynamic ports* …

Energy equations in the indicated sections $S_1$ and $S_2$ are :

$S_1$) $$E_{Tot} = p_{st} + \frac{1}{2}\rho v^2$$

$S_2$) $$E_{Tot} = p + \frac{1}{2}\rho w^2$$ where $p \neq p_{st}$ is the generic pressure

we get: $$p_{st} + \frac{1}{2}\rho v^2 = p + \frac{1}{2}\rho w^2$$

$$p - p_{st} = \frac{1}{2}\rho\left(v^2 - w^2\right)$$

$$p - p_{st} = \frac{1}{2}\rho v^2\left(1 - \frac{w^2}{v^2}\right)$$

where $K_p = 1 - \frac{w^2}{v^2} = \frac{2(p - p_{st})}{\rho v^2}$ is the ***pressure coefficient***

which is an important coefficient to decide "where to put the ports" on the probe !

***Kp*** is maximum (**= 1**) when ***w = 0***, or ***v = - $v_f$*** and the speed of the fluid element is slowed down to zero. This situation occurs only on the tip of the probe, along the probe axis. In that point the generic pressure ***p = $P_{tot}$*** is maximum. This is the right site to place the dynamic intake ***$P_d$***.

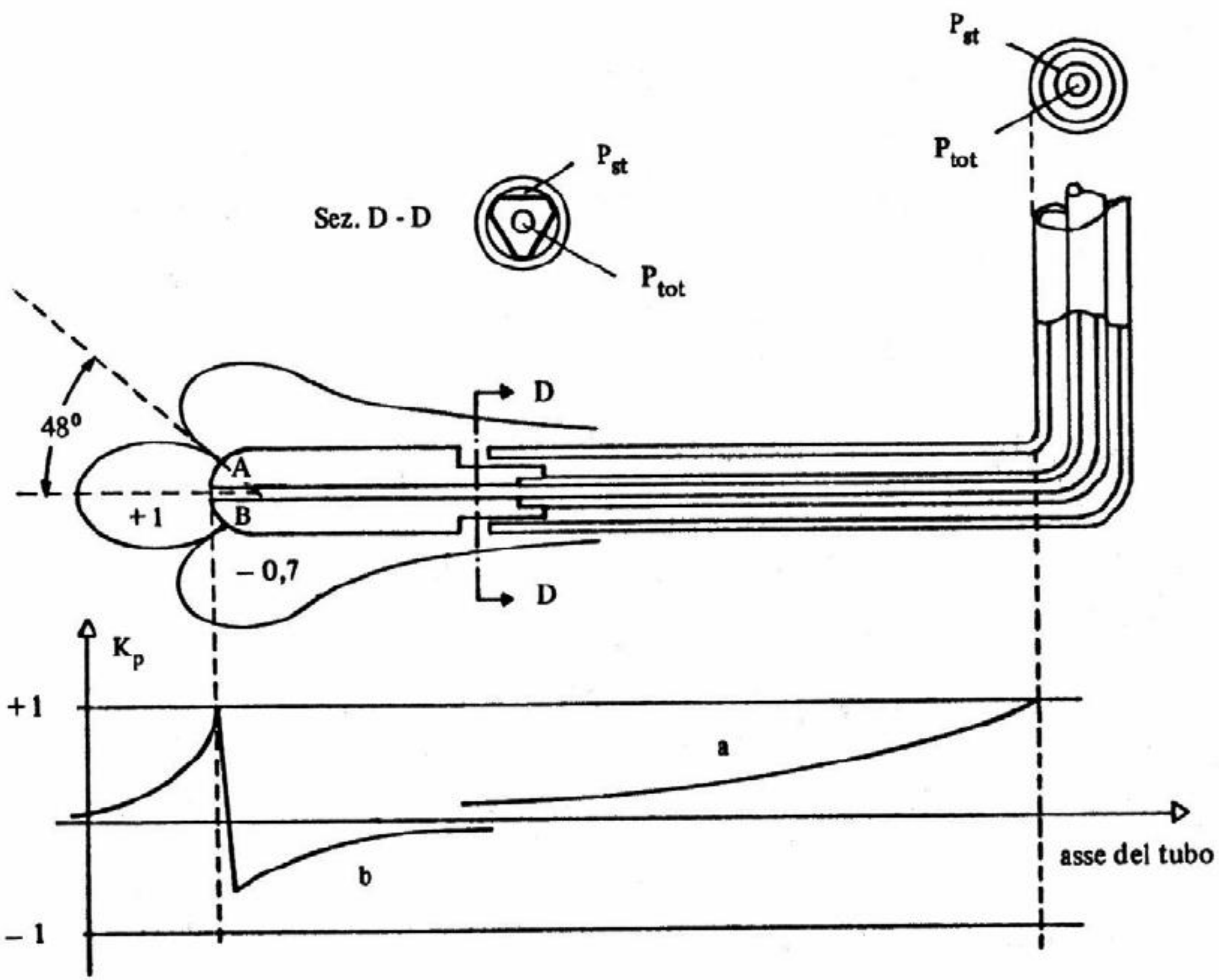
***Kp*** is null (**= 0**) when ***w = v***, or ***$v_f$ = 0*** where the surface of the tube does not cause any "interference" on the fluid element speed. In that point the generic pressure ***p = $p_{st}$*** is the static pressure. This is the right site to place the static port ***$P_s$***.

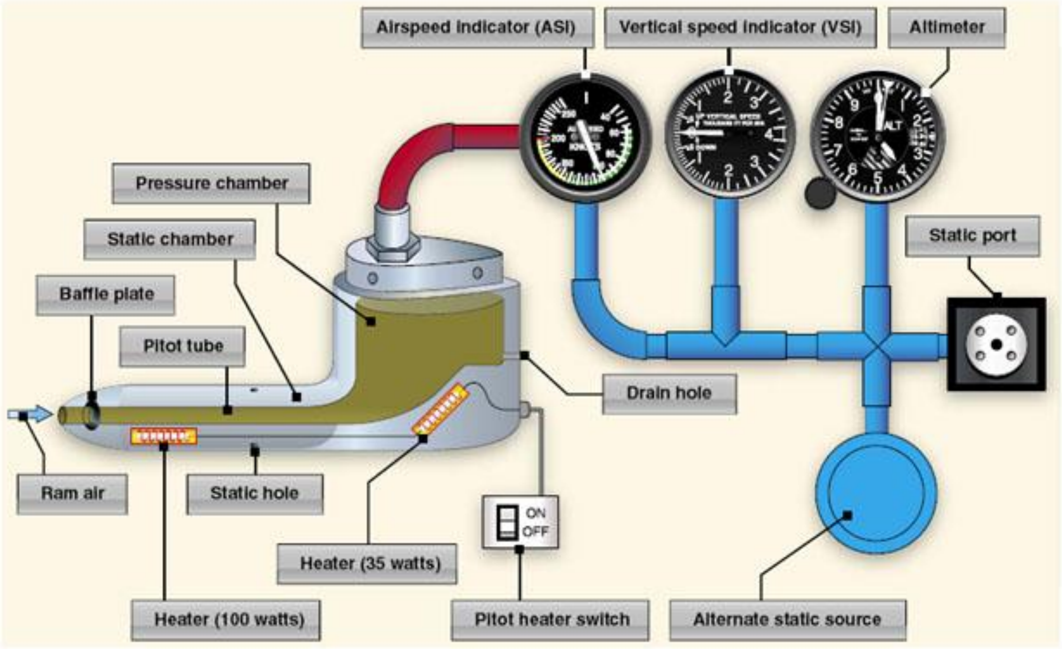
Note that the ***$K_p$*** can take on negative values in the tail regions of the probe, when ***w > v*** and ***p <$p_{st}$***, in the depression zones where the fluid accelerates to reform the undisturbed configuration of the fluid-dynamic field .

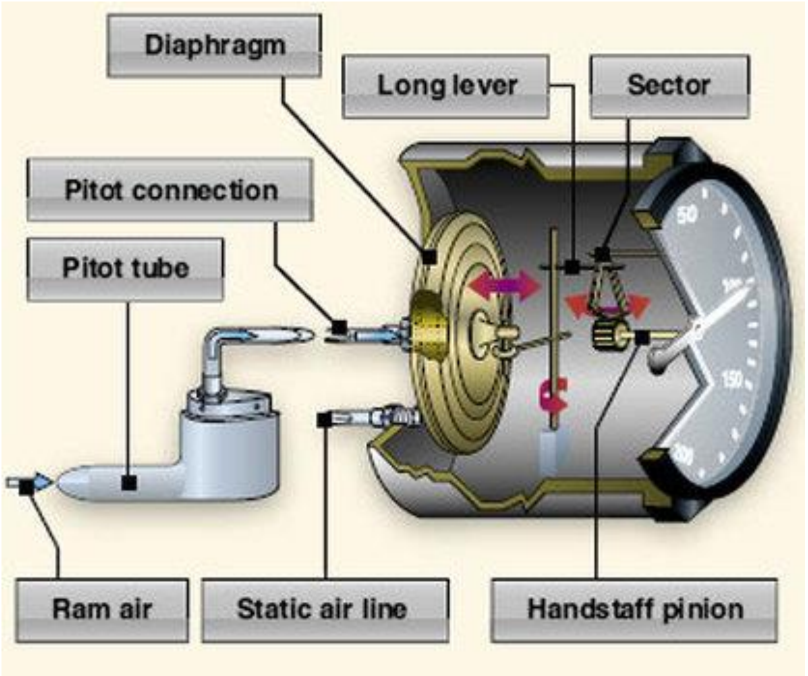
The *graduation curve* of such a probe is therefore:

$$p_{TOT} - p_{st} = \frac{1}{2}\rho v^2$$

# The real PITOT tube :



if we use the ***Pitot tube*** coupled with a ***water differential manometer*** to measure the velocity of air, in the graduation curve $v = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$ it is possible to measure directly the pressure just reading the $mmH_2O$ .

In fact, $1mmH_2O \cong 1kg_f / m^2$ and

if we express the density of air in the technical system:

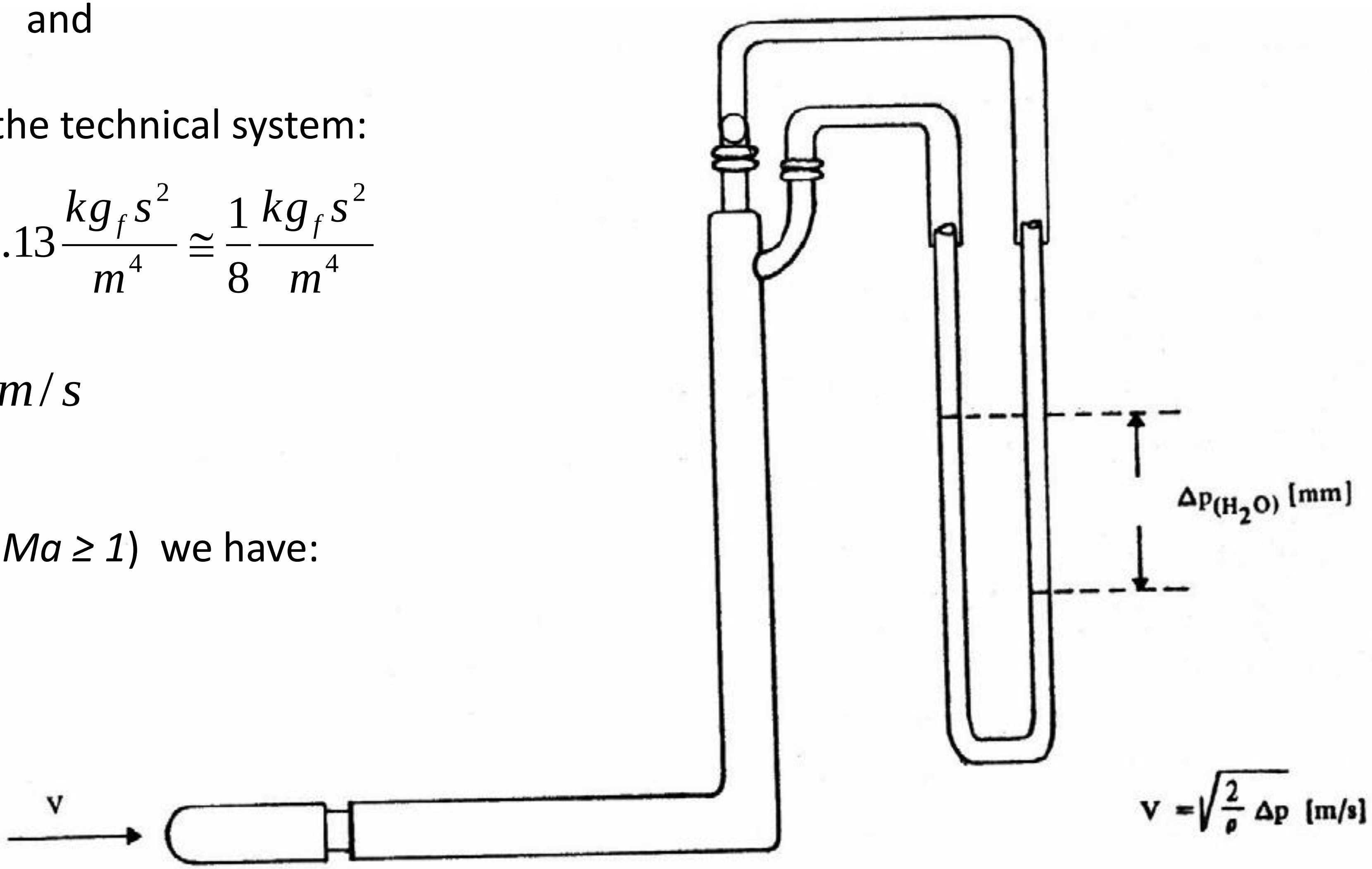
$$\rho_{aria} = 1.29 kg/m^3 = \frac{1.29}{9.81} \frac{M_p}{m^3} = 0.13 \frac{kg_f s^2}{m^4} \cong \frac{1}{8} \frac{kg_f s^2}{m^4}$$

we get $v = \sqrt{\frac{2}{1/8} \cdot 1} = \sqrt{16} = 4m/s$

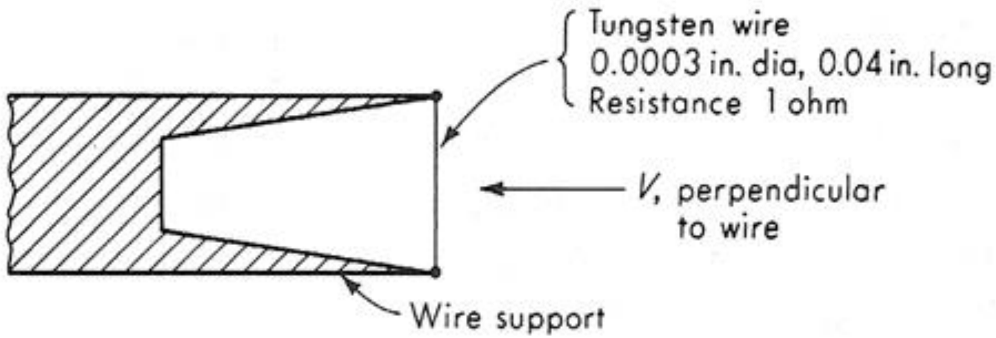
Note that for compressible fluids ($Ma \geq 1$) we have:

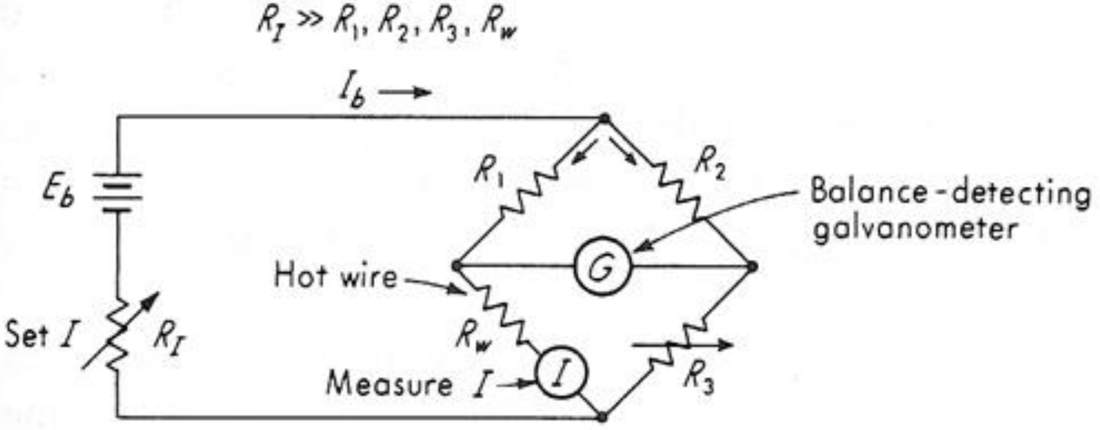
$$p_o = p + \frac{1}{2} \rho v^2 [1 + f(Ma)]$$

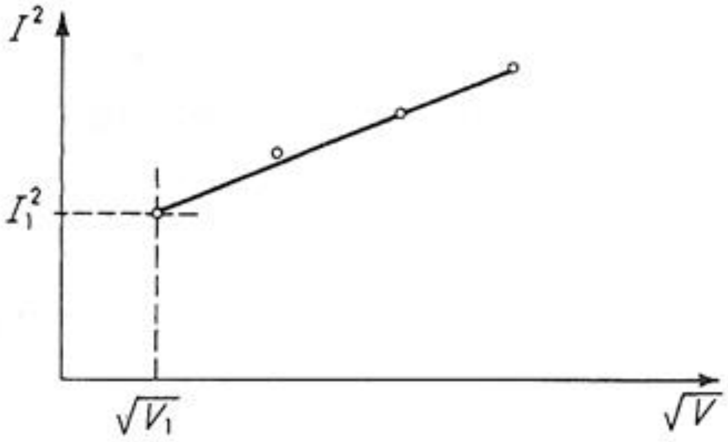
***Mach number***: $Ma = \frac{v}{c}$

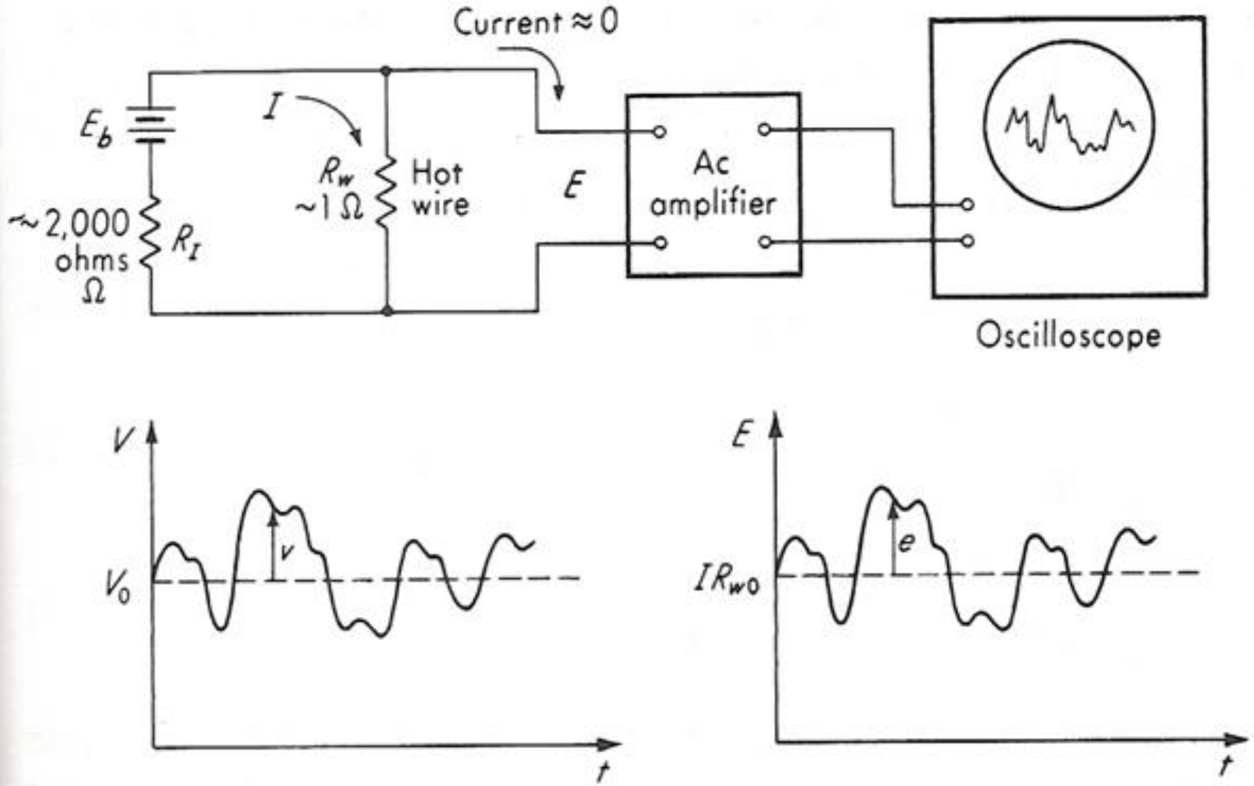


# Hot-wire anemometer



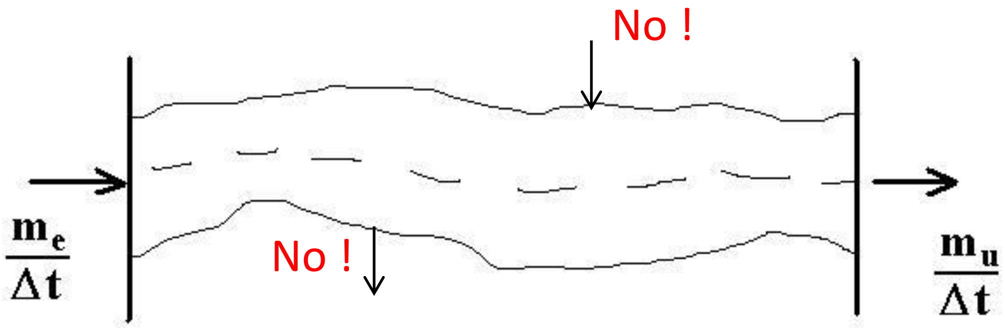




Velocity-fluctuation measurement.

***FLOW measurements*** are based on the principle of conservation of mass



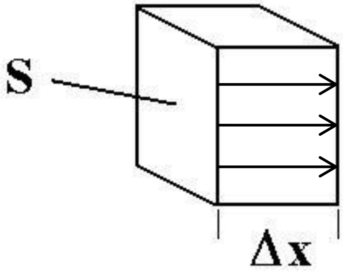

*Entering mass = outgoing mass* $(m_e = m_u)$

$$\left(\frac{\rho V}{\Delta t}\right)_e = \left(\frac{\rho V}{\Delta t}\right)_u$$

***Mass flow:*** $$Q_m = \frac{\rho V}{\Delta t}$$

***Weight flow rate:*** $$Q_p = \frac{\rho g V}{\Delta t}$$

***Volume flow rate:*** $$Q_v = \frac{V}{\Delta t}$$




Volume = $S \cdot \Delta x$ → $$\frac{S \cdot \Delta x}{\Delta t} = S \cdot v = Q_v$$

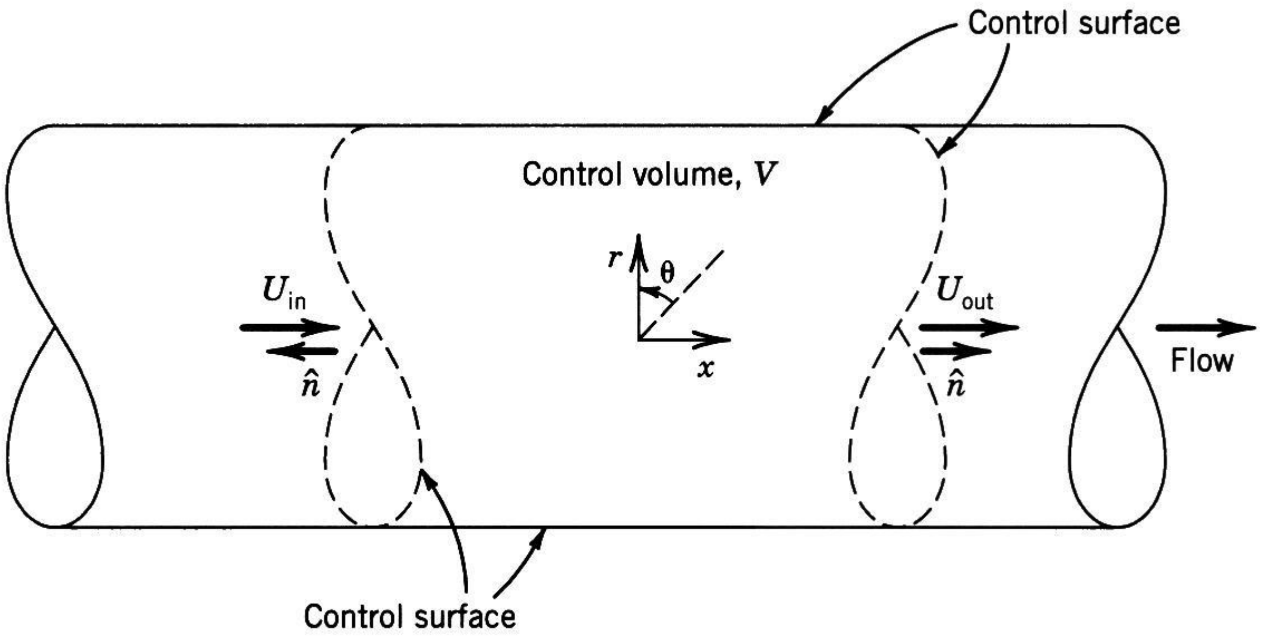
with $v = \frac{\Delta x}{\Delta t}$ velocity of the fluid "control" element !

*__Volumetric flow__* is always preferred when measuring flow in pipes :

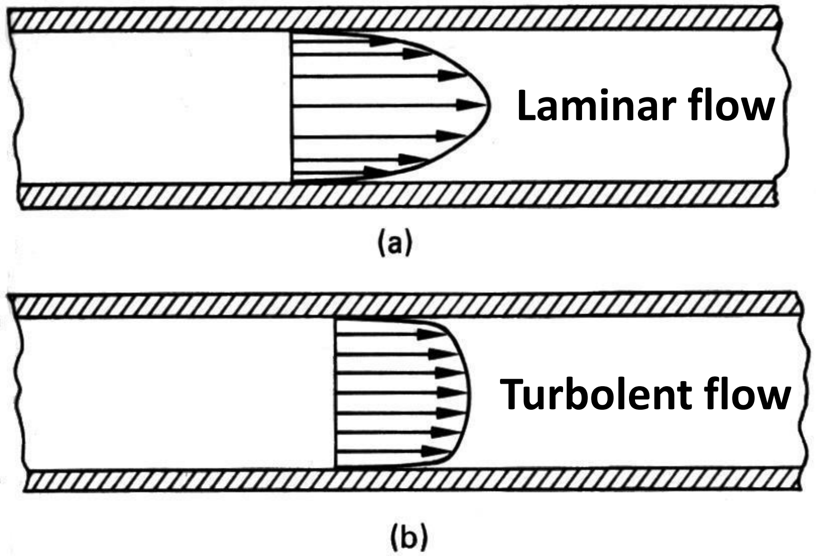
$$Q_v = S \cdot v$$

It's known !

?



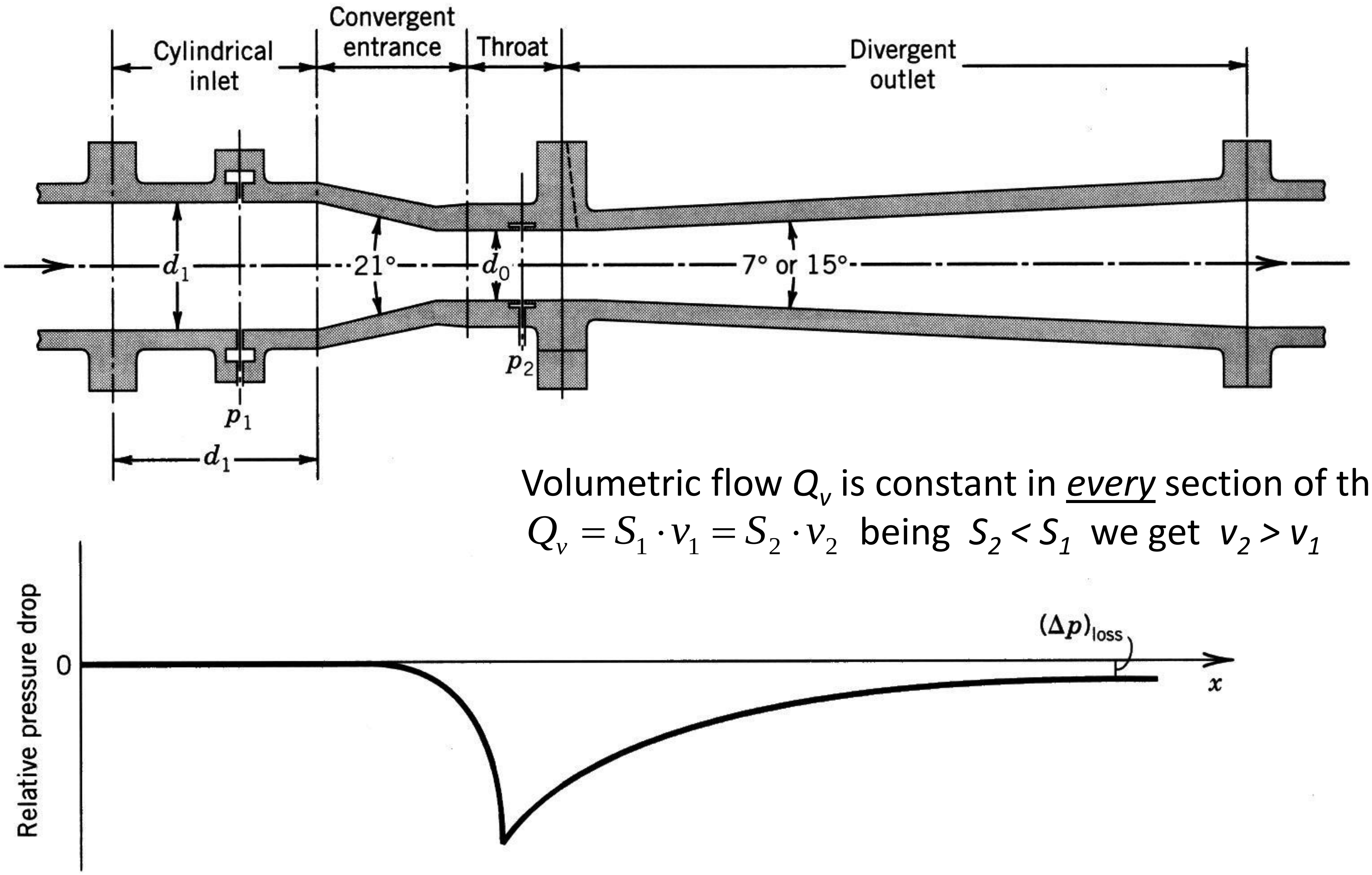

Control volume concept as applied to flow through a pipe.




In the case of ***incompressible fluid*** (*ρ = constant*), the speed distribution in the flow must be "measured" in some way and the average speed $\bar{v}$ must be calculated some how; only then we can write:

$$Q_v = S \cdot \bar{v} = \pi r^2 \cdot \bar{v}$$

For flow speed $\bar{v} < 3 \div 4 m/s$ pressure drop methods are employed: the ***VENTURI meter***




Volumetric flow $Q_v$ is constant in *every* section of the pipe

$Q_v = S_1 \cdot v_1 = S_2 \cdot v_2$ being $S_2 < S_1$ we get $v_2 > v_1$

Herschel venturi meter with the associated flow pressure drop along its axis.

The BERNOULLI theorem is applied between the sections $S_1$ and $S_2$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

The *throttling coefficient* is : $Z = \frac{S_2}{S_1} < 1$ and being $v_1 = \frac{S_2}{S_1} v_2$

$$v_1 = Z \cdot v_2$$

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2\left(1 - Z^2\right)$$

$$v_2^2 = \frac{2(p_1 - p_2)}{\rho\left(1 - Z^2\right)} \quad \rightarrow \quad v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1 - Z^2\right)}}$$

***Volumetric flow***:

$$Q_v = S_2 \cdot v_2 = S_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1 - Z^2\right)}}$$

Weight flow:

$$Q_p = \rho g S_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left(1 - Z^2\right)}} = g S_2 \sqrt{\frac{2\rho(p_1 - p_2)}{1 - Z^2}}$$

The losses (always present) are taken into account by a **discharge coefficient** :

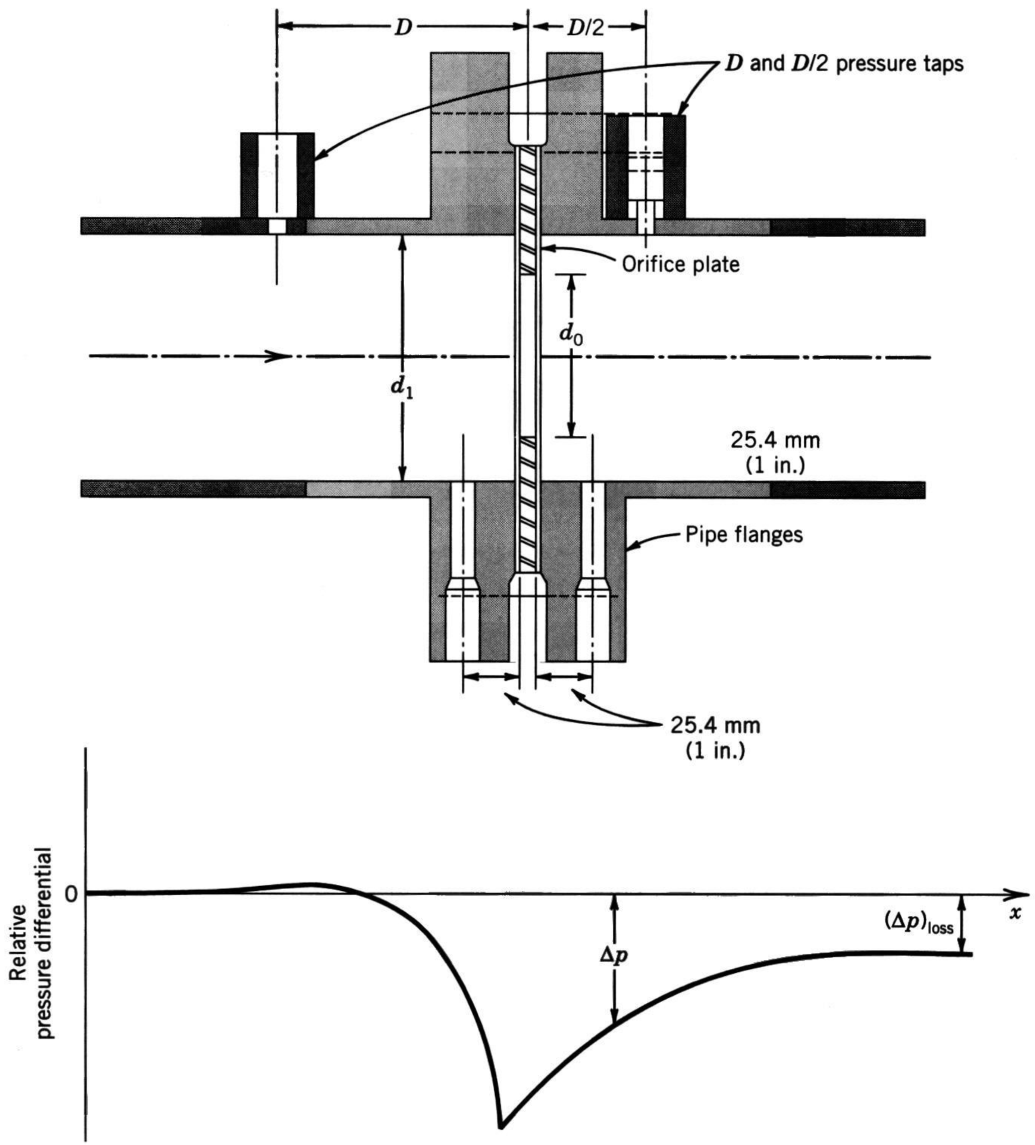
***Discharge coefficient:***

$$C = \frac{Q_{reale}}{Q_{ideale}} < 1$$

Actually, *C = C(Re)* is provided by the instrument producer !

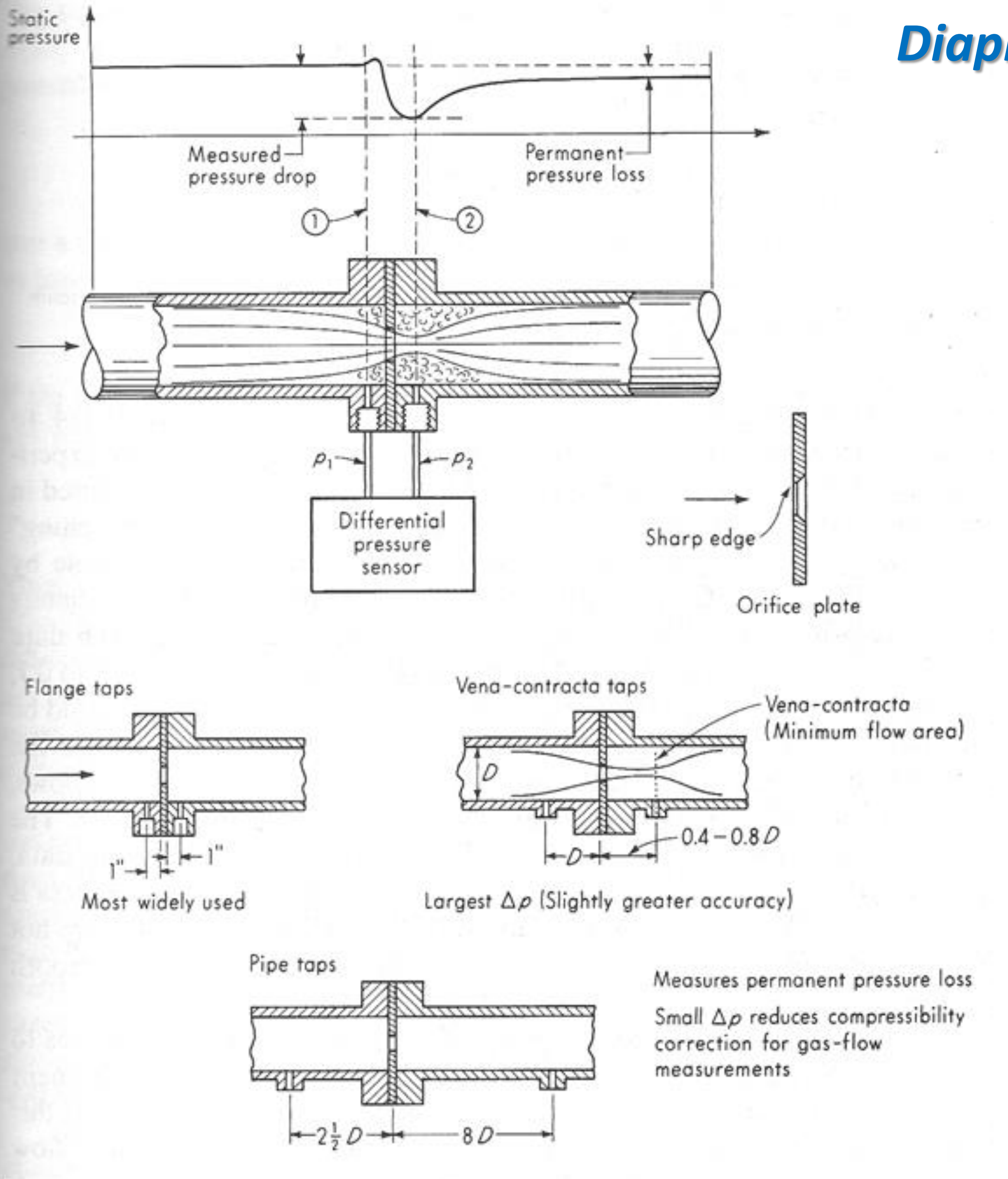
If we do not have at our disposal the necessary length to insert in the pipe a *Venturi flow meter,* we can employ a ***diaphragm flow meter***:

It has the same graduation curve of the Venturi meter but with a *much smaller discharge coefficient C = C (Re)* because of the considerable induced pressure losses (over 40%) happening at the diaphragm bottleneck !

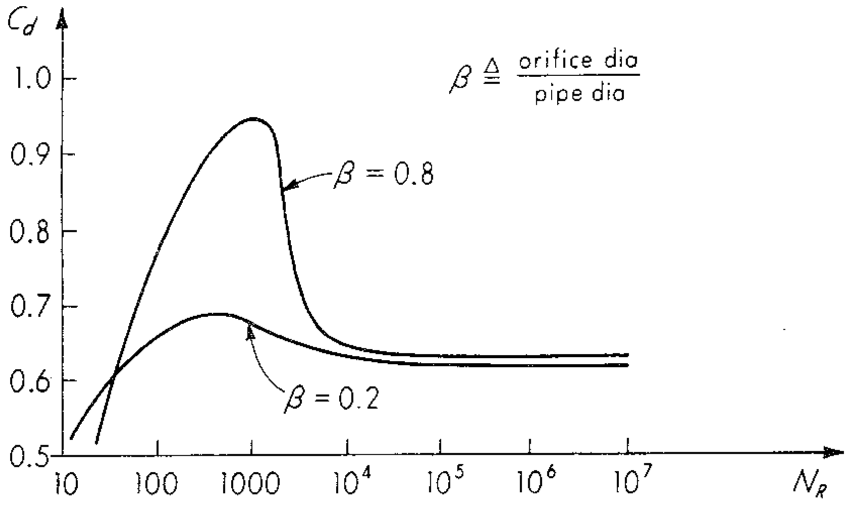



Square-edged orifice meter installed in a pipeline with optional one pipe and one-half pipe diameters, and flange pressure taps shown. Relative flow pressure drop along the pipe axis is shown.

# Diaphragm flow meter

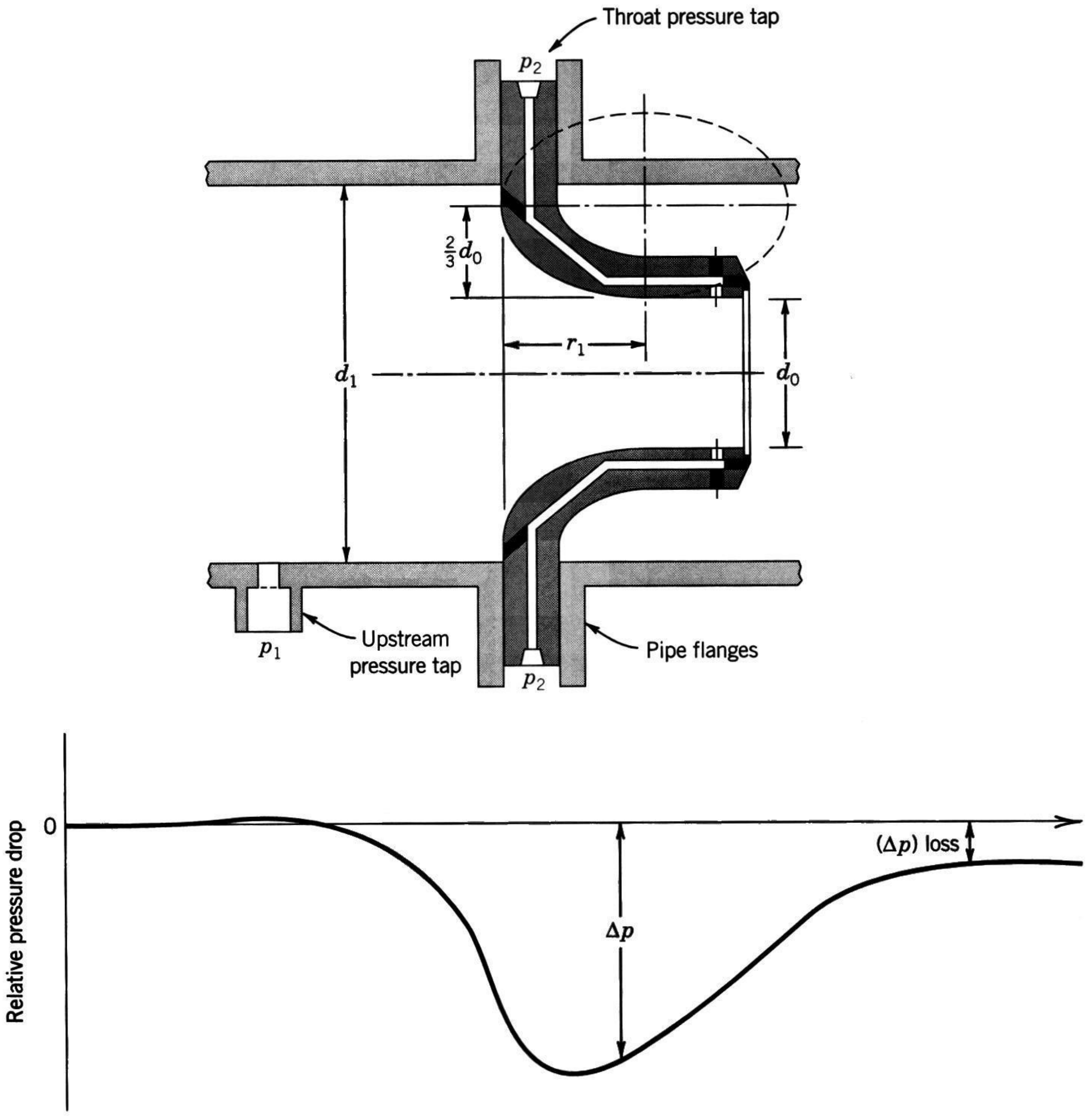



Orifice flowmetering.

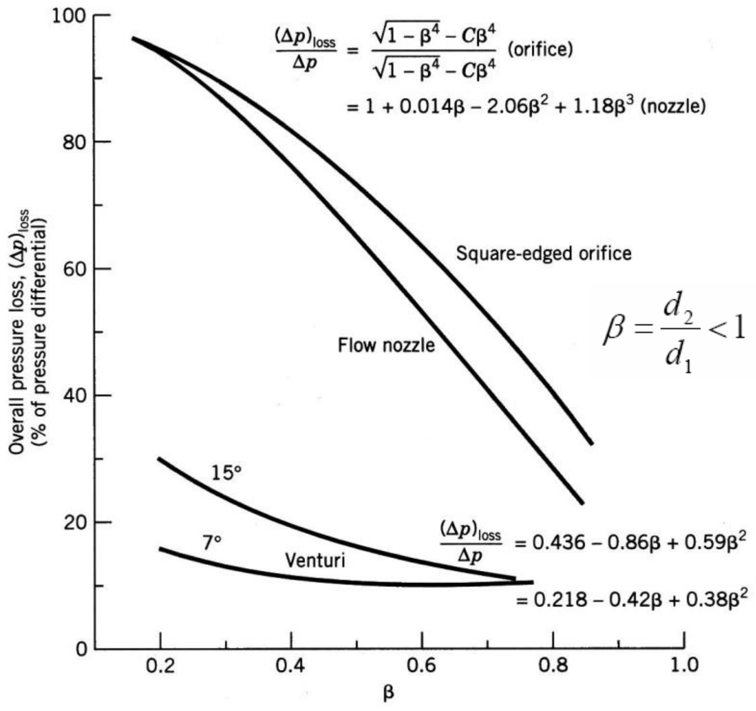




( $N_R$ = Re )

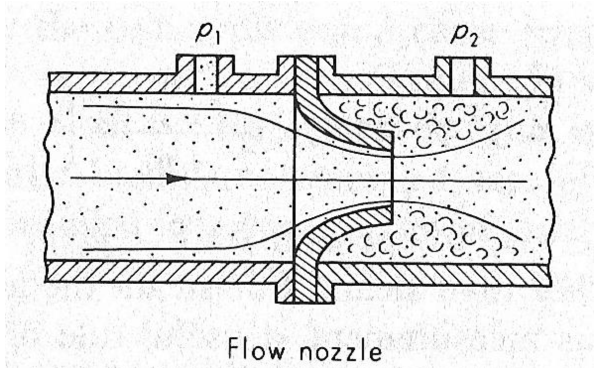
An intermediate solution can be the ***nozzle flow meter*** (with *pressure losses* around 20%):



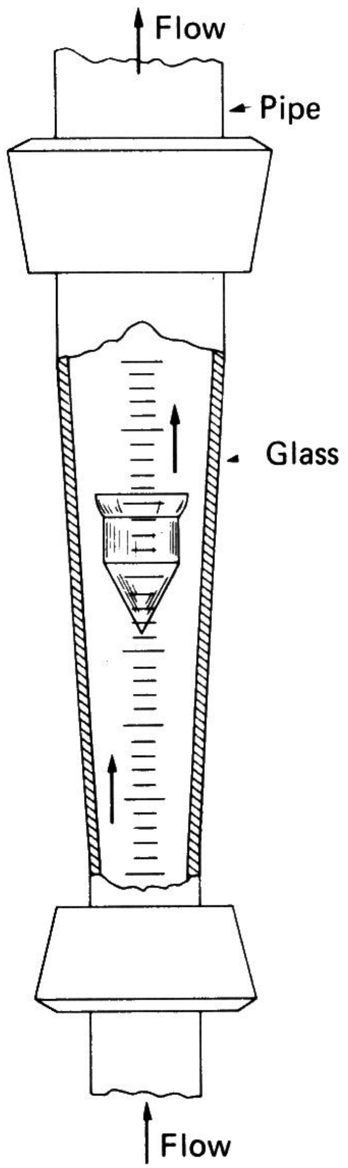

ASME long-radius nozzle with the associated flow pressure drop along its axis.




The permanent pressure loss associated with flow through common obstruction meters. (Courtesy of the American Society of Mechanical Engineers, New York)



# *Rotameter flow meter :*


The *Graduation curve* is based on the equilibrium of two forces:

1) weight (in water) of the floater: $\rho^* g V_c$

2) thrust (push-up) of the flow: $(p_1 - p_2)A$

with $\rho^* = \rho_c - \rho_f$ (floater density – fluid density)

$V_c$ floater volume and $A$ maximum floater cross section.

$$(\rho_c - \rho_f) g V_c = (p_1 - p_2)A$$

$$(\rho_c - \rho_f)\frac{gV_c}{A} = p_1 - p_2 \rightarrow \Delta p = \cos t$$

being all the quantities listed in the left term of the equation above "constant", the pressure difference between "below" and "above" the floater is also constant !

If we apply the **Bernoulli theorem** we get : $p_1 - p_2 = \frac{1}{2}\rho_f v^2$

***flow velocity** at the «annular section» which is always constant !*

$$\left(\rho_c - \rho_f\right)\frac{gV_c}{A} = \frac{1}{2}\rho_f v^2$$

$$v^2 = \frac{2gV_c}{A}\cdot\frac{\rho_c - \rho_f}{\rho_f}$$

$$v = \sqrt{\frac{2gV_c}{A}\cdot\frac{\rho_c - \rho_f}{\rho_f}}$$

If the *flow velocity v* is "constant" for each annular section, or for each height reached by the float, being $Q_v = S \cdot v$ when the flow *Q* changes there must necessarily be a "variation of the section *S*" ...
Therefore, we have to make the transparent tube with a "truncated cone shape" !

The graduation curve is :

$$Q_v = C \cdot S \cdot v = C \cdot S \cdot \sqrt{\frac{2gV_c}{A}\cdot\frac{\rho_c - \rho_f}{\rho_f}}$$

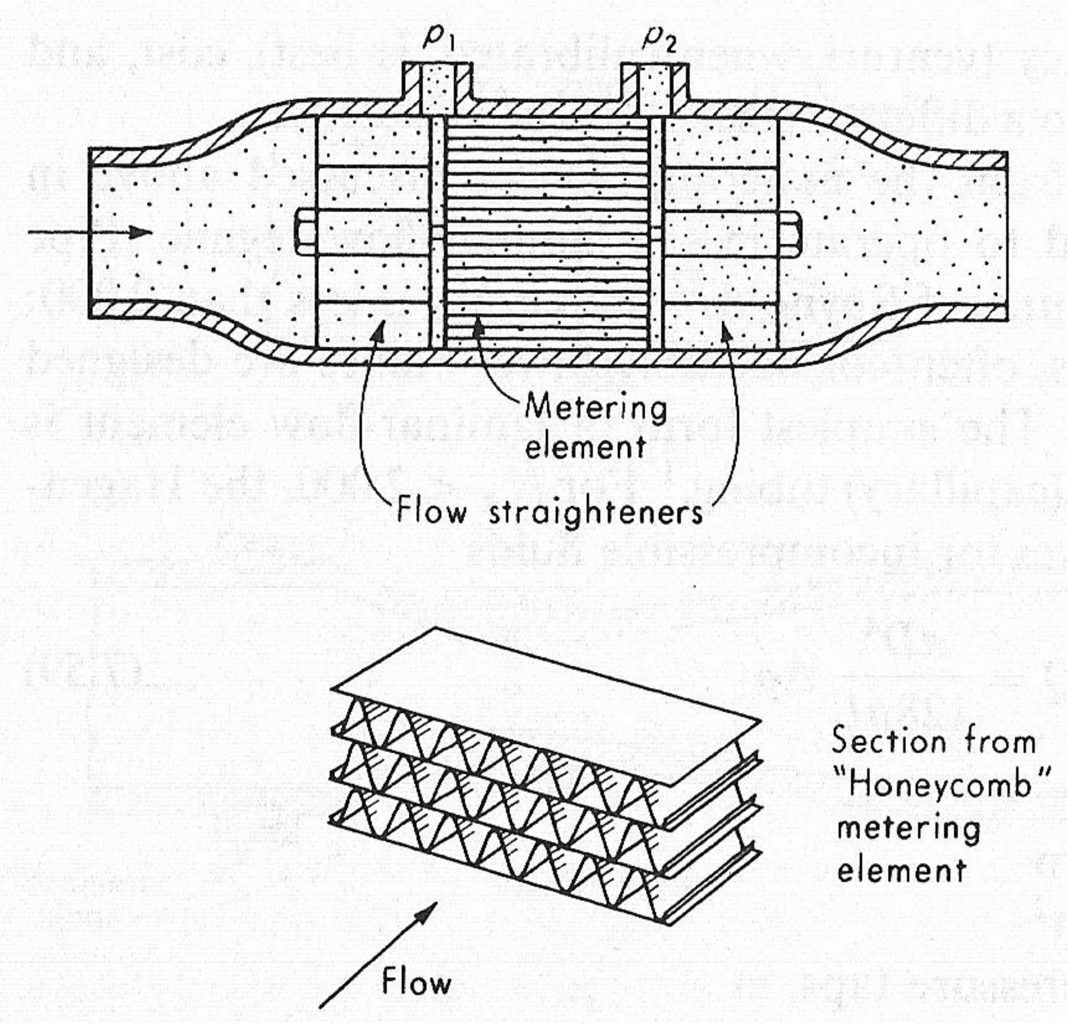
with $S = A_t(h) - A$

Note in the left figure that rotameter flow meter are often "coupled as a shunt" with bigger diaphragm flow meter ...

# *Laminar flow meter :*

- They are designed to "force" the flow to remain laminar!
- They work for fluid media with Re <2000
- They measure even very small flow rates from a few cc/h to several tens of $m^3$/min
- The *graduation curve* is the ***Hagen-Poiseuille law*** :

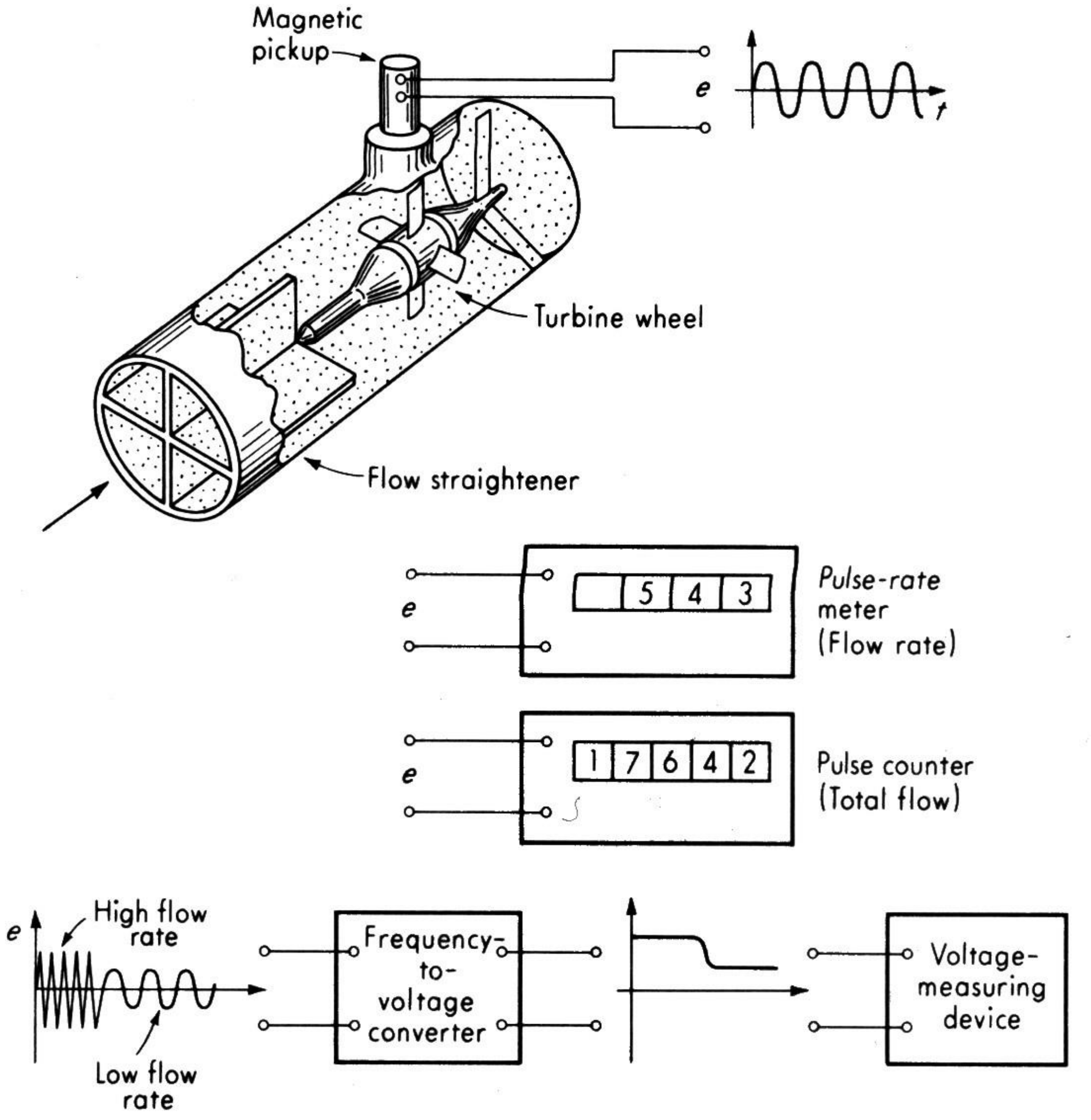
$$Q = \frac{\pi D^4}{128 \mu L} \cdot \Delta p$$



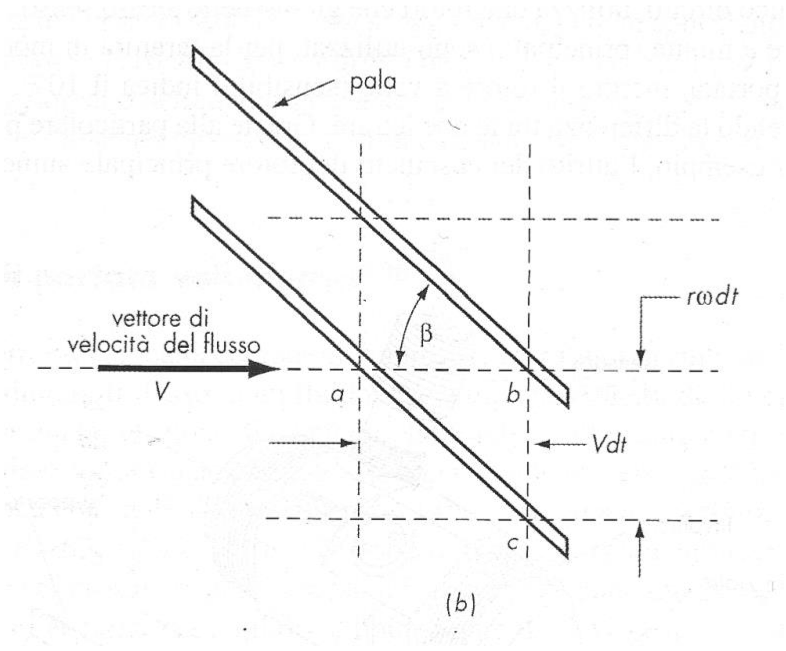

They are often used to measure gaseous flow (medical) at low pressure and non-stationary conditions !

PROS: linear scale between *Q* and *Δp*, insensitive to interference, bi-directional flows ...
AGAINST: introduce considerable *Δp*, becomes clogged if the gas is not clean ...

# *Electromechanical turbine flow meter:*




Turbine flowmeter.




In the hypothesis of "absence of friction and inertia", in every infinitesimal time ***dt*** the fluid goes from (a) to (b) while the vane goes from (c) to (b).
Therefore we can write the simple relationships:

$$\frac{r\omega \cdot dt}{v \cdot dt} = tg\beta$$

*Graduation curve*:

$$\mathrm{Q} = \mathrm{S} \cdot v = S \cdot \frac{r\omega}{tg\beta}$$

The **K factor** for the *ideal volumetric meter* (pulse num./liter), should be a constant.

In reality, especially for low flow rates, the rotation speed of the turbine is affected by the *viscosity* of the processed fluid (density and temperature).

There are "smart instruments" (with microprocessor) that manage to “compensate” this change (*span turndown*), however, only with fluids for which the calibration of the instrument was carried out ...

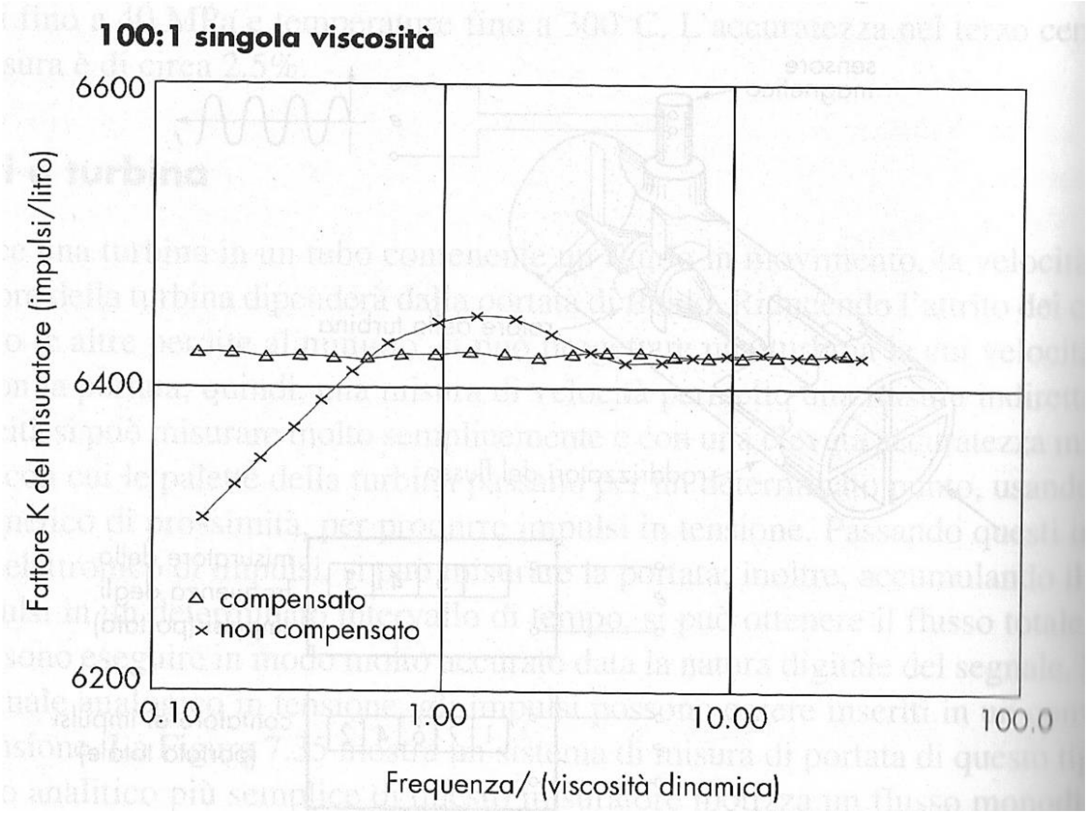
***Typical Specifications:***

*Span: 50 cc/min – 150 $m^3$/min (liqidi)*
*300 cc/min – 450 $m^3$/min (gas)*

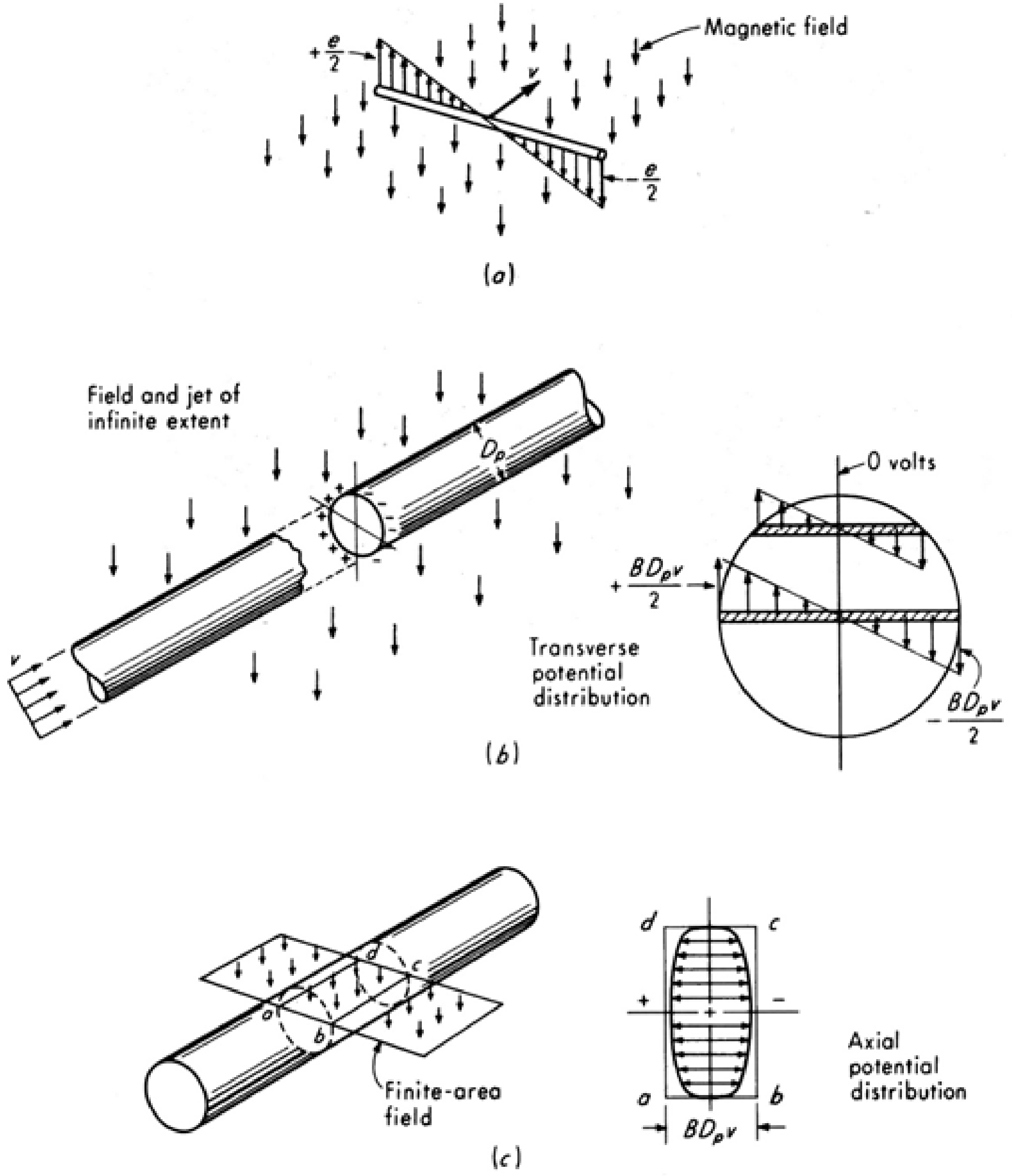
*non-linearity error = 0,1 %*

*Δp max = 200-700 hPa*

*1° order measuring system*
*(τ = 10 ms)*



# Electromagnetic flow meters :




Electromagnetic flowmeter.

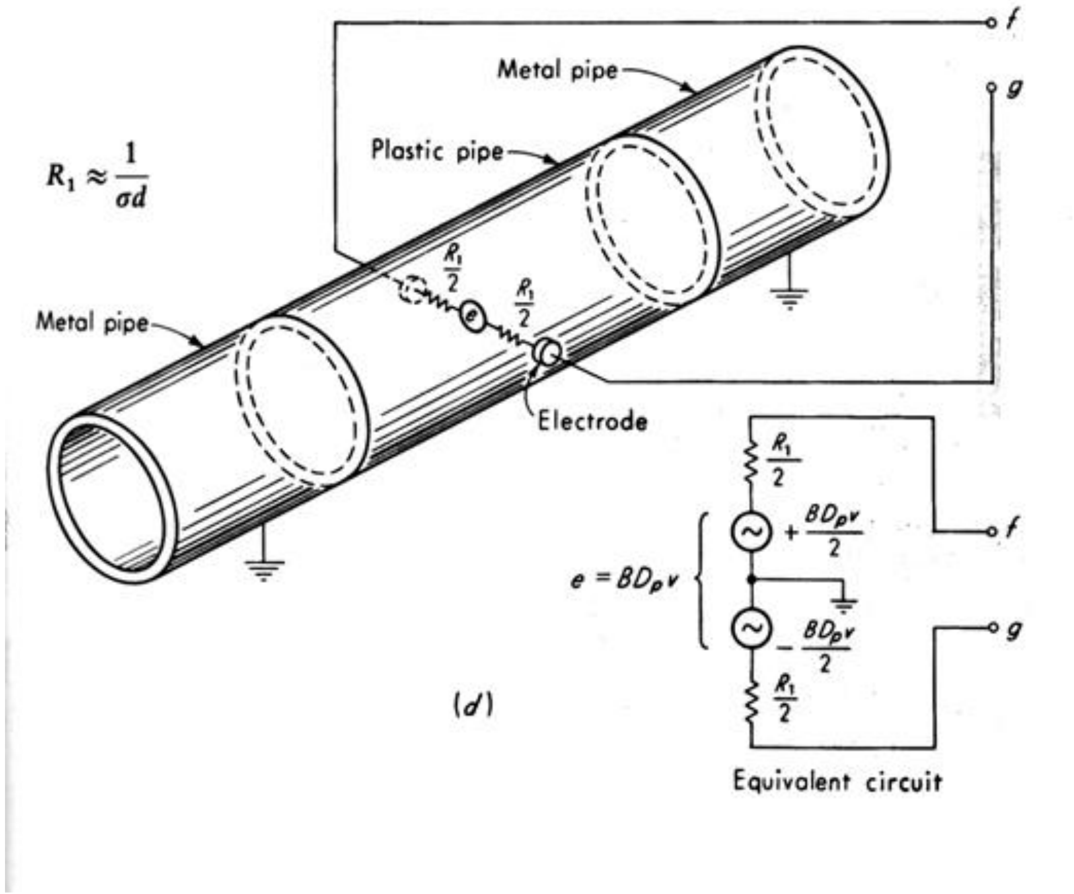
Based on the <u>electromagnetic induction</u> principle:

$$e = Blv$$

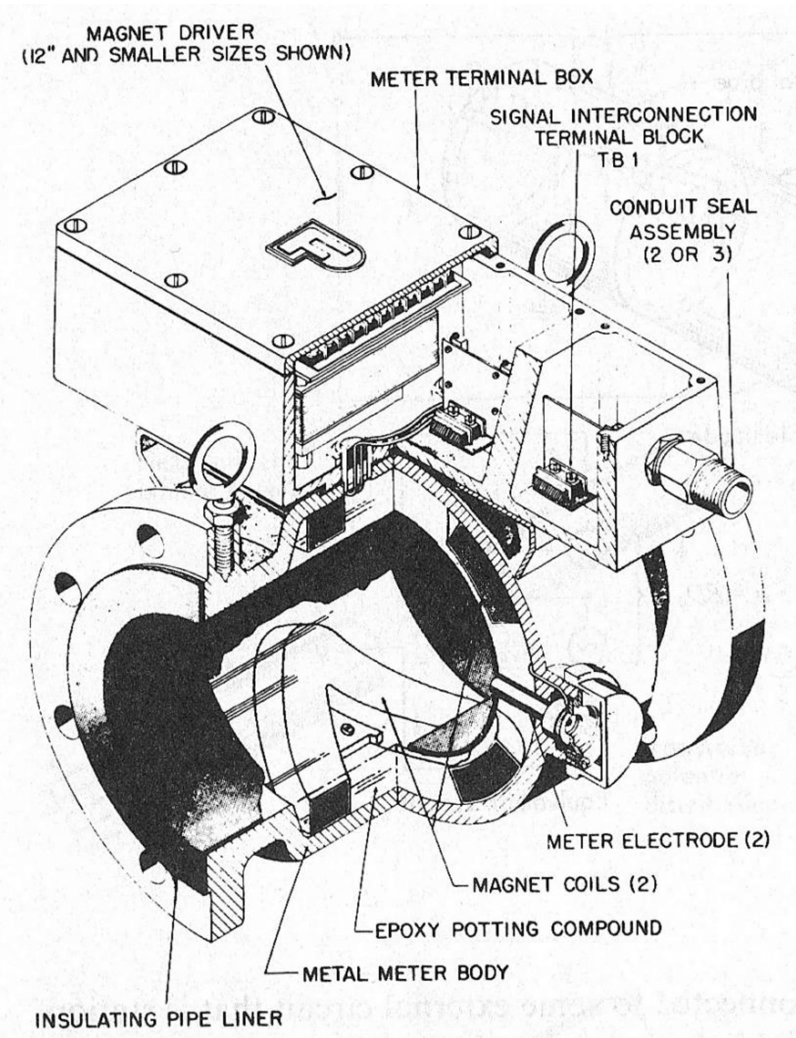
B → magnetic induction field [$Wb/m^2$]
l → moving electrical conductor [m]
v → velocity of the electrical conductor [m/s]

In a conductive fluid which passes through a magnetic field, there is a "ion separation" in the flow tube that generates a distribution of electrical potentials. The highest values are at the ends of the flow tube diameter: $BD_pv$

In real instruments, the area of the magnetic field B application is limited and the electric potential values measurable at the diameter point are smaller (*the fluid exposed to the magnetic field B tends to short-circuit the potential developed under the B field*)




If the extension of the B field is at least *3 diameters* $D_p$, the effect of the lateral short-circuit is reduced at the center of the B field; there are located the two electrodes to measure the electric potential !




The flowmeter tube must be made of NON magnetic material in the measurement zone, so as not to distort the field lines ***B*** …. and must be made of NON conductive material so as not to short circuit the electrical signal ***e*** !

The electric potential ***e = Blv*** causes a small current ***i*** between the electrodes, that passes through the conduction path of the fluid, which has a resistance ***R -> e = R · i***

***R*** can be estimated at ***1/σd*** (if *σ conductivity of the fluid* = 200 S/cm for water; *d diameter of the tube* = eg. 1 cm; then R = 5000 Ω) and must be measured during the calibration of the instrument.

***R*** determines the *load effect* on the signal manipulation circuit connected downstream ...

The *magnetic field* ***B*** can be applied either:

alternating "ac" (low polarization of the electrodes, low distortion of the velocity profiles, low signal shifts, stable amplification, but induction of an *ac spurious signals* on the measuring signal)

continuously switched "dc" (by square waves of a few Hz "auto-zeroing", slower than the ac instruments, with time constants $\tau = 2 - 6$ s )

The limit of the fluid conductivity is: ***0.1 mS/cm*** below, we don't get any electrical signal anymore !

This method is used for the measurement of blood flow in the capillaries with diameters up to d = 1mm

Note: they *do not introduce any obstruction to the flow* and are *insensitive to fluid density variations, viscosity and flow disturbances* ... as long as the profile of ***v*** remains symmetrical …

Additional problems may arise if the pipeline is not full, eg. for wastewater ...

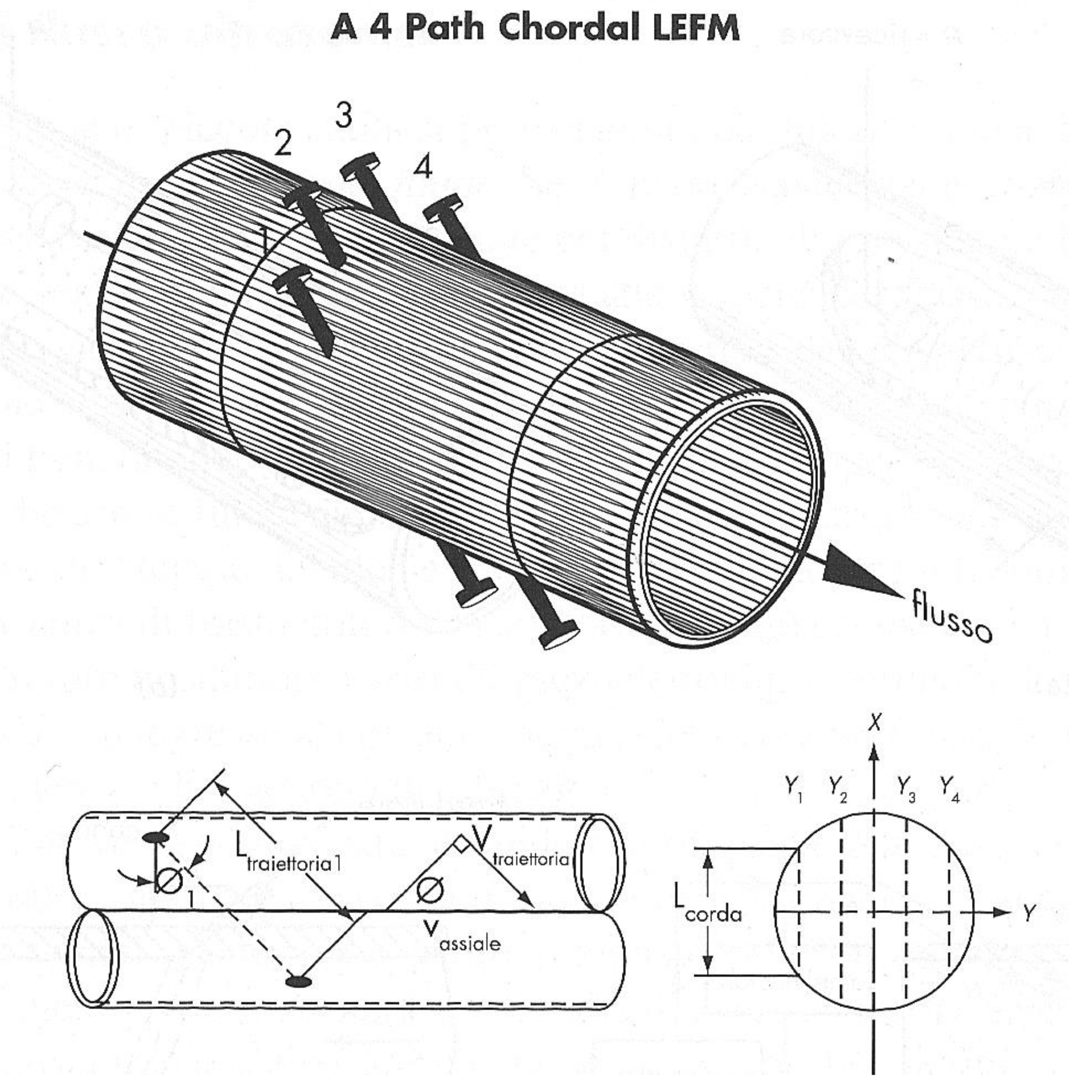
# *Ultrasound flow meters :*

The <u>sound</u> in a fluid (*a longitudinal pressure waves with appropriate frequency*) propagates with speed dependent on the <u>type</u> (E, ρ) and on the <u>conditions</u> (T) of the fluid itself !

If the fluid is in motion, the absolute velocity of the sound perturbation is the "algebraic sum" of its propagation velocity in the medium and that of the fluid itself.

An appropriate positioning of the <u>sound emitter</u> and <u>receiver</u> along the flow tube, may allow an accurate determination of the velocity v of the fluid and, therefore, of its volumetric flow rate

$Q = S \cdot v$



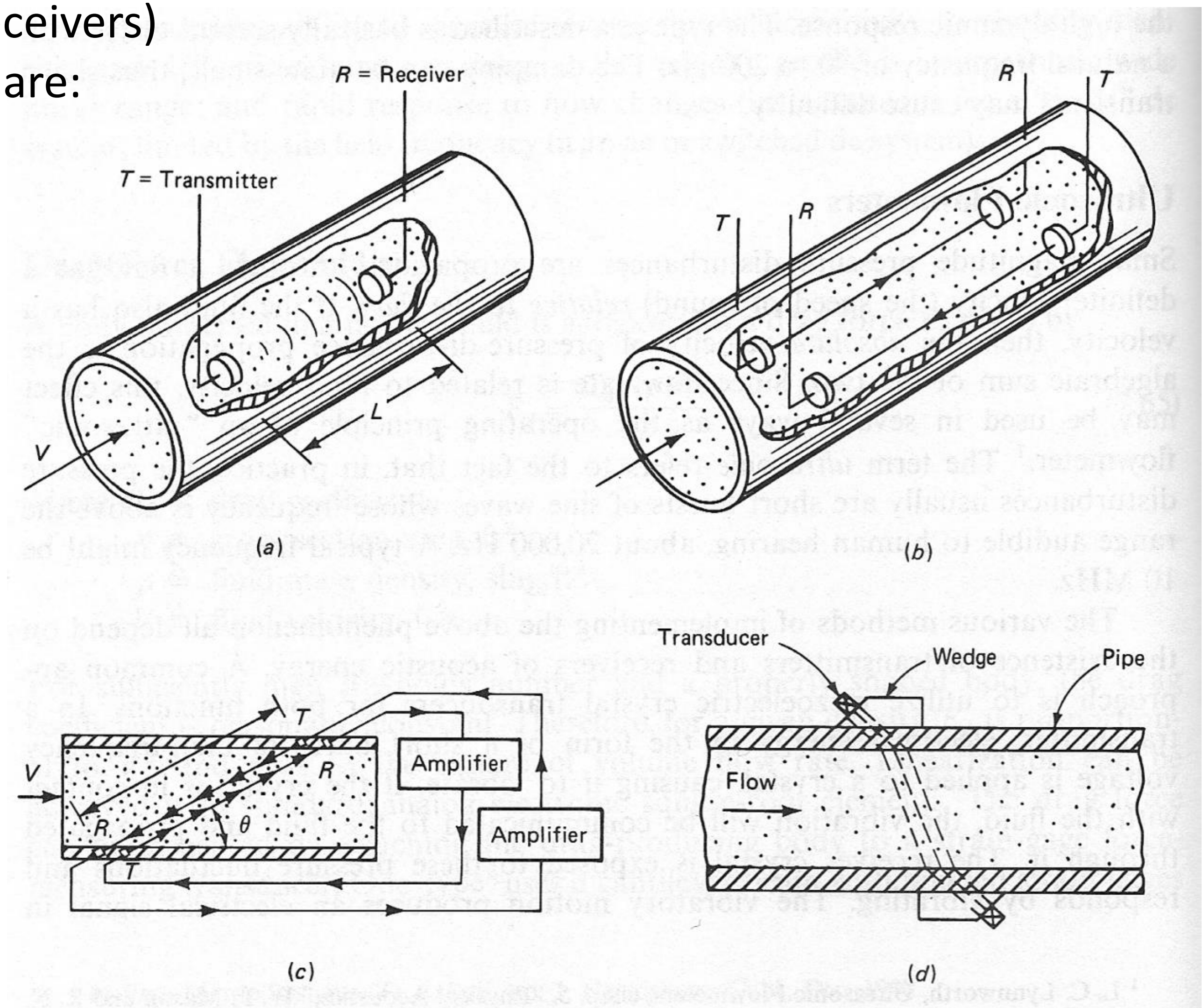

To obtain a tight and well defined acoustic field, ***ultrasonic wave packets*** are used with frequencies well above the audible (typically 10 MHz)

Transducers that are employed:

Almost only ***piezoelectric crystals*** (emitters/receivers)

The main *measurement techniques* employed are:

1. **flight time**
2. **doppler**




## *Flight time*

(a) with still fluid: $\boldsymbol{t_0 = \frac{L}{c}}$

for water c = 1520 m/s and if L = 30 cm the transit time is $t_0$ = 0,2 ms

with fluid in motion with a velocity *v* :

$$t = \frac{L}{c+v} \approx \frac{L}{c}\left(1 - \frac{V}{c}\right)$$

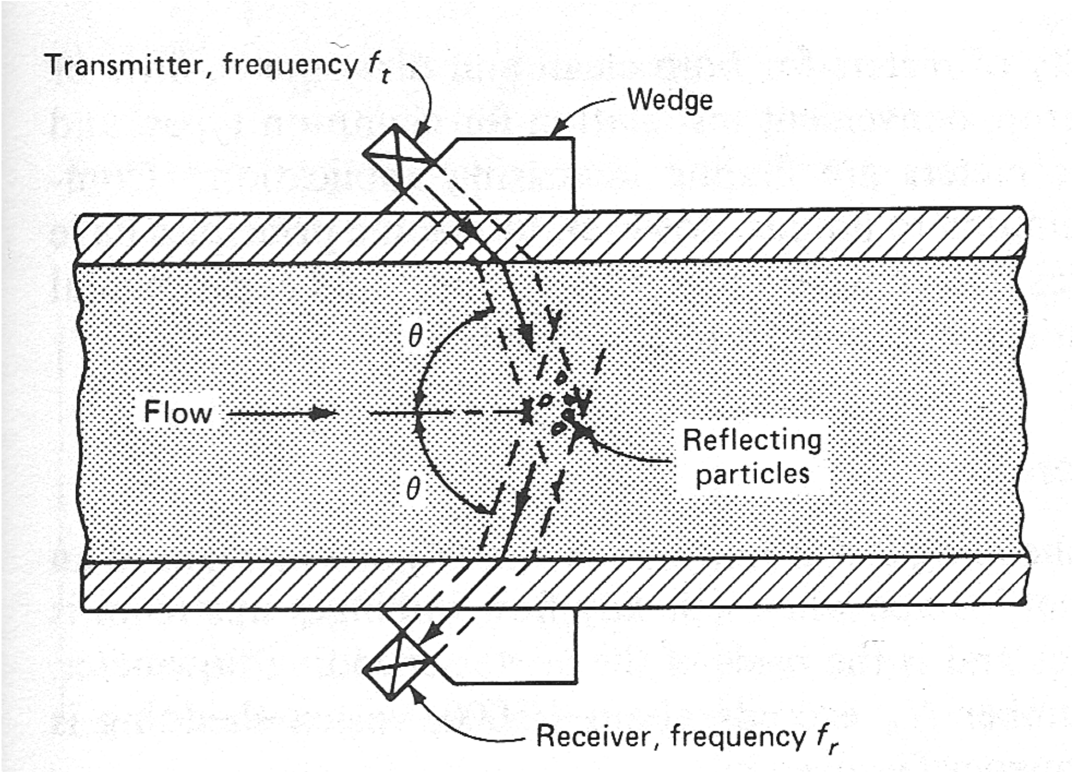
therefore $\boldsymbol{\Delta t = t_0 - t \approx \frac{Lv}{c^2}}$ generally small (a fraction of $\mu s$) and with $t_0$ not directly measurable

(b) $\boldsymbol{\Delta t = t_2 - t_1 \approx \frac{2Lv}{c^2}}$ with $t_1$ in the same direction of the flow and $t_2$ in the opposite direction, both measurable

(c) more complex $\Delta f = \frac{1}{t_f} - \frac{1}{t_b} = \frac{2v\cos\theta}{L}$

(d) with «clamp-on» transducers outside the pipe

**Doppler** (only for fluids with bubbles or with immersed particles:




The emitter sends an ***ultrasonic continuous wave*** (with frequency up to 10 MHz) that is reflected by the particles and measured by the receiver (we assume a uniform profile of *v*):

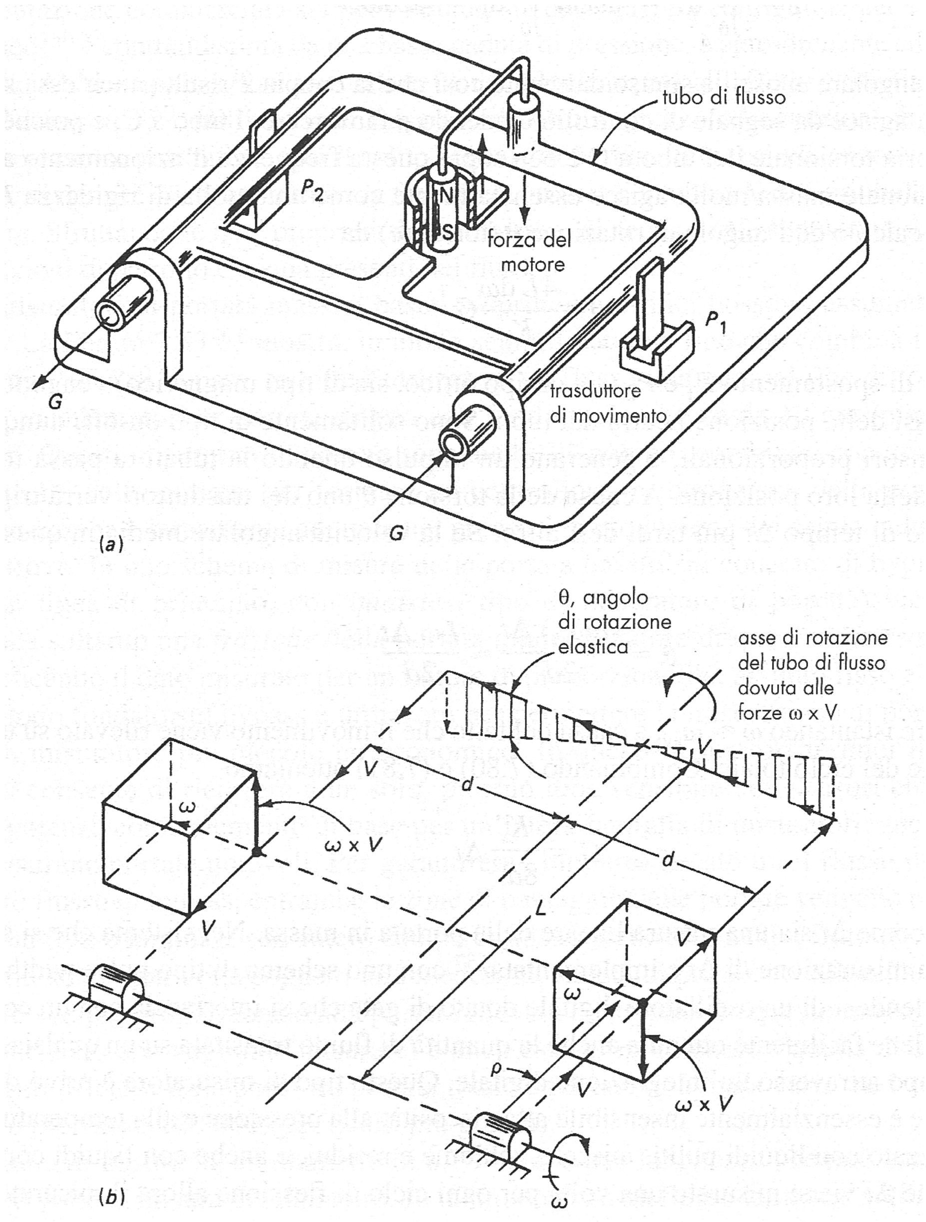
$$\Delta f = f_t - f_r = \frac{2 f_t \cos\theta}{c} \cdot v$$

Nonetheless, the real measurement is more complex and is based on FFT post processing …

Pulsed Doppler systems are commonly used in modern medical ultrasound systems for non-invasive measurement of flow in blood vessels ...

# To measure the flow of fuels or of chemical medium the ***mass flow ratio*** is more suited ...

$Q_m$ measurement through the **Coriolis effect**




$Q_m$ measurement by **heat transfer**